

المجموعة A تمارين مقالية

$$(1) F'(x) = 5(3x+2)^4 \times 3 = 15(3x+2)^4 = f(x)$$

إذاً F هي مشتقة عكسية للدالة f .

$$(2) F'(x) = x^2 - 2x + 1 = f(x)$$

إذاً F هي مشتقة عكسية للدالة f .

$$(3) F'(x) = \frac{1}{2}(1+x^4)^{-\frac{1}{2}} \times 4x^3 = \frac{2x^3}{\sqrt{1+x^4}} = f(x)$$

إذاً F هي مشتقة عكسية للدالة f .

$$(4) \int (x^5 - 6x + 3) dx = \frac{x^6}{6} - 3x^2 + 3x + C$$

$$(5) \int (3 - 6x^2) dx = 3x - 2x^3 + C$$

$$(6) \int \frac{1}{3} x^{-\frac{2}{3}} dx = x^{\frac{1}{3}} + C$$

$$(7) \int \left(x^3 - \frac{1}{x^3}\right) dx = \int (x^3 - x^{-3}) dx = \frac{x^4}{4} + \frac{x^{-2}}{2} + C = \frac{x^4}{4} + \frac{1}{2x^2} + C$$

$$(8) \int \frac{x^4 - 27x}{x^2 - 3x} dx = \int \frac{x^3 - 27}{x - 3} dx = \int (x^2 + 3x + 9) dx = \frac{x^3}{3} + \frac{3}{2}x^2 + 9x + C$$

$$(9) \int (x-2)(2x+3) dx = \int (2x^2 - x - 6) dx = \frac{2}{3}x^3 - \frac{x^2}{2} - 6x + C$$

$$(10) \int \frac{x-1}{\sqrt{x+1}} dx = \int \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x+1}} dx = \int (\sqrt{x}-1) dx = \frac{2}{3}x\sqrt{x} - x + C$$

$$(11) \int \frac{x-\sqrt{x}}{x} dx = \int \left(1 - \frac{1}{\sqrt{x}}\right) dx = x - 2\sqrt{x} + C$$

$$(12) \int \frac{5+2x}{\sqrt{x}} dx = \int \frac{5}{\sqrt{x}} dx + \int 2\sqrt{x} dx = 10\sqrt{x} + \frac{4}{3}x\sqrt{x} + C$$

$$(13) \int \left(x + \frac{1}{x}\right)^2 dx = \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx = \frac{x^3}{3} - \frac{1}{x} + 2x + C$$

$$(14) \int (3\sqrt{x^2} + 4\sqrt{x^3}) dx = \frac{3}{5}x^{\frac{5}{3}} + \frac{4}{7}x^{\frac{7}{4}} + C = \frac{3}{5}x\sqrt[3]{x^2} + \frac{4}{7}x\sqrt[4]{x^3} + C$$

$$(15) F(x) = x^3 - 5x + C$$

$$F(2) = 3 \quad \therefore \quad C = 5 \quad \therefore \quad F(x) = x^3 - 5x + 5$$

$$(16) F(x) = 3x^3 - 2x^2 + 5x + C$$

$$F(-1) = 0 \quad \therefore \quad C = 10 \quad \therefore \quad F(x) = 3x^3 - 2x^2 + 5x + 10$$

$$(17) r(x) = x^3 - 3x^2 + 12x + C$$

$$r(0) = 0 \quad \therefore \quad r(x) = x^3 - 3x^2 + 12x$$

(18) ليكن s ارتفاع الكرة فوق سطح الأرض عند الزمن t . نفرض أن s دالة في t قابلة للاشتقاق مرتين، ونرمز إلى سرعة

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad , \quad v = \frac{ds}{dt} : a \text{ بالرمز } v \text{ وإلى عجلتها بالرمز } a$$

(a) $a = -9.8$

$$a = \frac{dv}{dt} \implies -9.8 = \frac{dv}{dt}$$

$$v(t) = - \int 9.8 dt = -9.8t + C_1$$

$$16 = -9.8(0) + C_1$$

$$v(t) = -9.8t + 16$$

عندما تصل الكرة إلى أعلى ارتفاع، تكون $v(t) = 0$ ، أي أن:

$$-9.8t + 16 = 0 \quad \therefore \quad t = 1.63s$$

(b) $s(t) = \int v(t) dt = \int (-9.8t + 16) dt = -4.9t^2 + 16t + C_2$

$$s(0) = 115 \quad \therefore \quad C_2 = 115$$

$$s(t) = -4.9t^2 + 16t + 115$$

عندما تصل الكرة إلى الأرض يكون ارتفاعها $s(t) = 0$ ، أي أن:

$$-4.9t^2 + 16t + 115 = 0 \quad \therefore \quad t = 6.74s$$

المجموعة B تمارين موضوعية

- | | | | |
|---------|----------|----------|----------|
| (1) (a) | (2) (a) | (3) (b) | (4) (b) |
| (5) (b) | (6) (b) | (7) (a) | (8) (c) |
| (9) (c) | (10) (a) | (11) (b) | (12) (d) |

تمرن 2-5

التكامل بالتعويض

المجموعة A تمارين مقالية

(1) $u = x^2 - 3x + 5 \quad , \quad du = (2x - 3)dx$

$$\int (2x - 3)\sqrt{x^2 - 3x + 5} dx = \int u^{\frac{1}{2}} du = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(x^2 - 3x + 5)^{\frac{3}{2}} + C$$

(2) $u = 4x - 5 \quad , \quad du = 4dx$

$$\int (4x - 5)^8 dx = \int \frac{1}{4}u^8 du = \frac{u^9}{36} + C = \frac{(4x - 5)^9}{36} + C$$

(3) $u = x^2 + 4x - 1 \quad , \quad du = (2x + 4)dx = 2(x + 2)dx$

$$\int (x + 2)^3 \sqrt{x^2 + 4x - 1} dx = \int \frac{1}{2}u^{\frac{1}{2}} du = \frac{3}{8}u^{\frac{4}{3}} + C = \frac{3}{8}(x^2 + 4x - 1)^{\frac{4}{3}} + C$$

$$(4) \quad u = x^3 - 3x + 5, \quad du = (3x^2 - 3)dx = 3(x^2 - 1)dx$$

$$\int (x^2 - 1)\sqrt{x^3 - 3x + 5} dx = \int \frac{1}{3}u^{\frac{1}{2}} du = \frac{2}{9}u^{\frac{3}{2}} + C = \frac{2}{9}(x^3 - 3x + 5)^{\frac{3}{2}} + C$$

$$(5) \quad u = x^3 - 3x^2 + 4, \quad du = (3x^2 - 6x)dx = 3(x^2 - 2x)dx$$

$$\int (x^2 - 2x)(x^3 - 3x^2 + 4)^5 dx = \int \frac{1}{3}u^5 du = \frac{u^6}{18} + C = \frac{(x^3 - 3x^2 + 4)^6}{18} + C$$

$$(6) \quad u = 4 + x^3, \quad du = 3x^2 dx$$

$$\int \frac{x^2}{\sqrt[3]{4 + x^3}} dx = \int x^2(4 + x^3)^{-\frac{1}{3}} dx = \int \frac{1}{3}u^{-\frac{1}{3}} du = \frac{u^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{(4 + x^3)^{\frac{2}{3}}}{2} + C$$

$$(7) \quad u = 2 - 3x, \quad du = -3dx$$

$$\int \frac{dx}{\sqrt[3]{2 - 3x}} = \int (2 - 3x)^{-\frac{1}{3}} dx = \int -\frac{1}{3}u^{-\frac{1}{3}} du = -\frac{u^{\frac{2}{3}}}{\frac{2}{3}} + C = -\frac{(2 - 3x)^{\frac{2}{3}}}{2} + C$$

$$(8) \quad u = 3x + 2, \quad du = 3dx, \quad x = \frac{u}{3} - \frac{2}{3}$$

$$\begin{aligned} \int x(3x + 2)^6 dx &= \int \left(\frac{u}{3} - \frac{2}{3}\right)u^6 \times \frac{1}{3} du = \frac{1}{3} \left[\frac{u^8}{24} - \frac{2u^7}{21} \right] + C \\ &= \frac{u^8}{72} - \frac{2u^7}{63} + C = \frac{(3x + 2)^8}{72} - \frac{2(3x + 2)^7}{63} + C \end{aligned}$$

$$(9) \quad u = 1 + 3x, \quad du = 3dx, \quad x = \frac{u}{3} - \frac{1}{3}$$

$$\begin{aligned} \int \frac{x}{\sqrt{1 + 3x}} dx &= \int x(1 + 3x)^{-\frac{1}{2}} dx = \int \left(\frac{u}{3} - \frac{1}{3}\right)u^{-\frac{1}{2}} \times \frac{1}{3} du = \frac{1}{9} \int (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) du \\ &= \frac{2}{27}u^{\frac{3}{2}} - \frac{2}{9}u^{\frac{1}{2}} + C = \frac{2}{27}(1 + 3x)^{\frac{3}{2}} - \frac{2}{9}(1 + 3x)^{\frac{1}{2}} + C \end{aligned}$$

$$(10) \quad u = x - 1, \quad du = dx, \quad x^2 = (u + 1)^2$$

$$\begin{aligned} \int x^2 \sqrt{x - 1} dx &= \int (u + 1)^2 \times u^{\frac{1}{2}} \times du = \int (u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}}) du \\ &= \frac{2}{7}u^{\frac{7}{2}} + \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{7}(x - 1)^{\frac{7}{2}} + \frac{4}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + C \end{aligned}$$

$$(11) \quad u = x^2 - 2, \quad du = 2x dx, \quad x^2 = u + 2$$

$$\begin{aligned} \int x^2 \cdot x \sqrt{x^2 - 2} dx &= \frac{1}{2} \int (u + 2)u^{\frac{1}{2}} du = \frac{1}{2} \int u^{\frac{3}{2}} du + \int u^{\frac{1}{2}} du \\ &= \frac{1}{5}(x^2 - 2)^{\frac{5}{2}} + \frac{2}{3}(x^2 - 2)^{\frac{3}{2}} + C \end{aligned}$$

$$(12) \quad u = x^3 + 1, \quad du = 3x^2 dx, \quad x^3 = u - 1$$

$$\begin{aligned} \int x^3 \cdot x^2 (x^3 + 1)^{\frac{1}{3}} dx &= \frac{1}{3} \int (u - 1) \times u^{\frac{1}{3}} du = \frac{1}{3} \int u^{\frac{4}{3}} du - \frac{1}{3} \int u^{\frac{1}{3}} du \\ &= \frac{1}{7}(x^3 + 1)^{\frac{7}{3}} - \frac{1}{4}(x^3 + 1)^{\frac{4}{3}} + C \end{aligned}$$

المجموعة B تمارين موضوعية

- | | | | |
|---------|----------|----------|----------|
| (1) (b) | (2) (a) | (3) (b) | (4) (a) |
| (5) (a) | (6) (a) | (7) (b) | (8) (d) |
| (9) (b) | (10) (a) | (11) (c) | (12) (b) |

تمرن 3-5

تكامل الدوال المثلثية

المجموعة A تمارين مقالية

- (1) $\int (\sec x \tan x + \sin x) dx = \sec x - \cos x + C$
- (2) $\int (\csc x \cot x + \sec^2 x) dx = -\int -\csc x \cot x dx + \int \sec^2 x dx = -\csc x + \tan x + C$
- (3) $\int \left(-\frac{1}{x^2} + 5 \sin 3x\right) dx = \frac{1}{x} - \frac{5}{3} \cos 3x + C$
- (4) $\int \sin^4 x \cos x dx = \frac{\sin^5 x}{5} + C$
- (5) $\int \cos^5 x \sin x dx = -\frac{\cos^6 x}{6} + C$
- (6) $\int x^2 \sin(x^3 + 1) dx = \frac{1}{3} \int 3x^2 \sin(x^3 + 1) dx = -\frac{1}{3} \cos(x^3 + 1) + C$
- (7) $\int \frac{\sin x}{\cos^3 x} dx = \int \sin x (\cos x)^{-3} dx = -\frac{(\cos x)^{-2}}{-2} + C = \frac{1}{2} \sec^2 x + C$
- (8) $\int \sec^3 x \tan x dx = \int \sin x \times (\cos x)^{-4} dx = -\frac{(\cos x)^{-3}}{-3} + C = \frac{1}{3} \sec^3 x + C$
- (9) $\int \csc^3 x \cot x dx = \int \cos x \times (\sin x)^{-4} dx = \frac{(\sin x)^{-3}}{-3} + C = -\frac{1}{3} \csc^3 x + C$
- (10) $\int \sqrt{\cot x} \csc^2 x dx = -\int \sqrt{\cot x} (-\csc^2 x) dx = -\frac{2}{3} \cot^{\frac{3}{2}} x + C$
- (11) $\int \sqrt{\tan x} \sec^2 x dx = \frac{2}{3} \tan^{\frac{3}{2}} x + C$
- (12) $\int \sqrt{1 + \sin x} \cos x dx = \frac{2}{3} (1 + \sin x)^{\frac{3}{2}} + C$
- (13) $\int \frac{1}{(\sin^2 x) \sqrt{1 + \cot x}} dx = -\int \frac{-1}{\sin^2 x} (1 + \cot x)^{-\frac{1}{2}} dx = -2\sqrt{1 + \cot x} + C$
- (14) $\int \frac{1}{(\cos^2 x) \sqrt{1 + \tan x}} dx = \int \frac{1}{\cos^2 x} (1 + \tan x)^{-\frac{1}{2}} dx = 2\sqrt{1 + \tan x} + C$

المجموعة B تمارين موضوعية

- (1) (a) (2) (b) (3) (b) (4) (a)
(5) (a) (6) (c) (7) (d) (8) (b)
(9) (c) (10) (c) (11) (b) (12) (b)

تمرن 4-5

الدوال الأسية واللوغاريتمية

المجموعة A تمارين مقالية

- (1) $\frac{dy}{dx} = (\ln 7) \times 7^x$
(2) $\frac{dy}{dx} = \frac{\ln 5}{2\sqrt{x+1}} \times 5^{\sqrt{x+1}}$
(3) $\frac{dy}{dx} = (\ln 8)(\sec^2 x) \times 8^{\tan x}$
(4) $\frac{dy}{dx} = 2e^x$
(5) $\frac{dy}{dx} = -e^{-x}$
(6) $\frac{dy}{dx} = \frac{3}{5}e^{\frac{x}{5}}$
(7) $\frac{dy}{dx} = (2x-1)e^{x^2-x+1}$
(8) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}e^{2\sqrt{x}+3}$
(9) $\frac{dy}{dx} = -\csc x \cot x \cdot e^{\csc x}$
(10) $\frac{dy}{dx} = 4x^3 e^{x^4-5}$
(11) $\frac{dy}{dx} = \frac{3}{x}$
(12) $\frac{dy}{dx} = -\frac{2}{x}$
(13) $\frac{dy}{dx} = \frac{1}{x+2}$
(14) $\frac{dy}{dx} = \frac{\sin x}{2-\cos x}$
(15) $\frac{dy}{dx} = \frac{1}{x \ln x}$
(16) $\frac{e^{0.1x}}{0.1} + C = 10e^{0.1x} + C$

- (17) $-e^{\frac{1}{x}} + C$
 (18) $e^{x^2+x+4} + C$
 (19) $\frac{1}{3}e^{x^3-6x} + C$
 (20) $\frac{1}{0.5}e^{0.5x} + 0.5\ln|x| + C$
 (21) $\ln(e^x + 1) + C$
 (22) $\frac{1}{2}\ln(x^2 + 2x + 5) + C$
 (23) $\frac{1}{4}\ln|x^4 - 2x^2| + C$
 (24) $\frac{x^2}{2} + \ln|x| + C$
 (25) $\frac{2}{3}\ln|3x + 1| + C$
 (26) $-2\ln|\cos x| + \cot x + C$
 (27) $\ln|\sin x| + \frac{x^3}{3} + C$

المجموعة B تمارين موضوعية

- (1) (b) (2) (b) (3) (b) (4) (a)
 (5) (b) (6) (b) (7) (c) (8) (a)
 (9) (b) (10) (d) (11) (c) (12) (b)
 (13) (a) (14) (b)

تمرن 5-5

التكامل بالتجزئ

المجموعة A تمارين مقالية

- (1) $u = x$ $dv = \cos(3x) dx$
 $du = dx$ $v = \frac{\sin(3x)}{3}$
 $\int x \cos(3x) dx = \frac{x}{3} \sin(3x) - \frac{1}{3} \int \sin(3x) dx$
 $= \frac{x}{3} \sin(3x) + \frac{1}{9} \cos(3x) + C$
- (2) $u = x$ $dv = \sin(5x) dx$
 $du = dx$ $v = -\frac{1}{5} \cos(5x)$
 $\int x \sin(5x) dx = -\frac{x}{5} \cos(5x) + \frac{1}{5} \int \cos(5x) dx$
 $= -\frac{x}{5} \cos(5x) + \frac{1}{25} \sin(5x) + C$

$$(3) \quad u = x, \quad du = dx$$

$$dv = e^{x-3} dx \quad v = e^{x-3}$$

$$\int x e^{x-3} dx = x e^{x-3} - \int e^{x-3} dx = x e^{x-3} - e^{x-3} + C$$

$$(4) \quad \int (x-5)e^{x-5} dx = (x-6)e^{x-5} + C$$

$$(u = x-5, \quad dv = e^{x-5} dx \text{ : إرشاد})$$

$$(5) \quad u = \ln \sqrt[4]{x} = \ln x^{\frac{1}{4}} = \frac{1}{4} \ln x \quad du = \frac{1}{4x} dx$$

$$dv = dx \quad v = x$$

$$\int \ln \sqrt[4]{x} dx = \frac{x}{4} \ln x - \int \frac{1}{4x} \times x dx = \frac{1}{4}(x \ln x - x) + C$$

$$(6) \quad u = \ln(2x-1) \quad du = \frac{2}{2x-1} dx$$

$$dv = dx \quad v = x$$

$$\begin{aligned} \int \ln(2x-1) dx &= x \ln(2x-1) - \int \frac{2x}{2x-1} dx = x \ln(2x-1) - \int \frac{2x-1+1}{2x-1} dx \\ &= x \ln(2x-1) - \int \left(1 + \frac{1}{2x-1}\right) dx \\ &= x \ln(2x-1) - x - \frac{1}{2} \ln(2x-1) + C \end{aligned}$$

$$(7) \quad \int (2x+1) \ln(x+1) dx = (x^2+x) \ln(x+1) - \frac{x^2}{2} + C \quad (u = \ln(x+1), \quad dv = (2x+1) dx \text{ : إرشاد})$$

$$(8) \quad u = \ln x \quad du = \frac{1}{x} dx$$

$$dv = \frac{1}{x^2} dx \quad v = -\frac{1}{x}$$

$$\int \frac{1}{x^2} \ln x dx = \frac{-\ln x}{x} + \int \frac{1}{x^2} dx = \frac{-\ln x}{x} - \frac{1}{x} + C$$

$$(9) \quad u = \ln x \quad du = \frac{1}{x} dx$$

$$dv = x^{-\frac{1}{3}} dx \quad v = \frac{3}{2} x^{\frac{2}{3}}$$

$$\begin{aligned} \int x^{-\frac{1}{3}} \ln x dx &= \frac{3}{2} x^{\frac{2}{3}} \ln x - \int \frac{1}{x} \times \frac{3}{2} x^{\frac{2}{3}} dx \\ &= \frac{3}{2} x^{\frac{2}{3}} \ln x - \frac{3}{2} \int x^{-\frac{1}{3}} dx = \frac{3}{2} \sqrt[3]{x^2} \left(\ln x - \frac{3}{2} \right) + C \end{aligned}$$

$$(10) \quad u = \ln x^2 = 2 \ln x \quad du = \frac{2}{x} dx$$

$$dv = x^2 dx \quad v = \frac{x^3}{3}$$

$$\int x^2 \ln x^2 dx = \frac{1}{3} x^3 \ln x^2 - \frac{2}{3} \int \frac{1}{x} \times x^3 dx = \frac{1}{3} x^3 \ln x^2 - \frac{2}{9} x^3 + C$$

$$(11) \quad u = x^2 - 2x \quad du = 2(x-1) dx$$

$$dv = \cos x dx \quad v = \sin x$$

$$\int (x^2 - 2x) \cos x \, dx = (x^2 - 2x) \sin x - 2 \int (x - 1) \sin x \, dx$$

$\int (x - 1) \sin x \, dx$ نستخدم القاعدة مرّة ثانية لإيجاد

$$u = x - 1 \quad du = dx$$

$$dv = \sin x \, dx \quad v = -\cos x$$

$$\int (x^2 - 2x) \cos x \, dx = (x^2 - 2x) \sin x - 2 \left[-(x - 1) \cos x + \int \cos x \, dx \right]$$

$$= (x^2 - 2x - 2) \sin x + 2(x - 1) \cos x + C$$

$$(12) \quad u = x^2 + 3x \quad du = (2x + 3) dx$$

$$dv = \sin x \, dx \quad v = -\cos x$$

$$\int (x^2 + 3x) \sin x \, dx = -(x^2 + 3x) \cos x + \int (2x + 3) \cos x \, dx$$

$\int (2x + 3) \cos x \, dx$ نستخدم القاعدة مرّة ثانية لإيجاد

$$u = 2x + 3 \quad du = 2 dx$$

$$dv = \cos x \, dx \quad v = \sin x$$

$$\int (x^2 + 3x) \sin x \, dx = -(x^2 + 3x) \cos x + (2x + 3) \sin x - 2 \int \sin x \, dx$$

$$= -(x^2 + 3x - 2) \cos x + (2x + 3) \sin x + C$$

$$(13) \quad u = x^2 \quad du = 2x \, dx$$

$$dv = e^{x+1} \, dx \quad v = e^{x+1}$$

$$\int x^2 e^{x+1} \, dx = x^2 e^{x+1} - \int 2x e^{x+1} \, dx$$

$\int x e^{x+1} \, dx$ نستخدم القاعدة مرّة ثانية لإيجاد

$$u = x \quad du = dx$$

$$dv = e^{x+1} \, dx \quad v = e^{x+1}$$

$$\int x^2 e^{x+1} \, dx = x^2 e^{x+1} - 2x e^{x+1} + 2 \int e^{x+1} \, dx = e^{x+1} (x^2 - 2x + 2) + C$$

$$(14) \quad u = x^2 \quad du = 2x \, dx$$

$$dv = e^{2x-3} \, dx \quad v = \frac{1}{2} e^{2x-3}$$

$$\int x^2 e^{2x-3} \, dx = \frac{x^2}{2} e^{2x-3} - \int x e^{2x-3} \, dx$$

$\int x e^{2x-3} \, dx$ نستخدم القاعدة مرّة ثانية لإيجاد

$$u = x \quad du = dx$$

$$dv = e^{2x-3} dx \quad v = \frac{1}{2} e^{2x-3}$$

$$\begin{aligned} \int x^2 e^{2x-3} dx &= \frac{x^2}{2} e^{2x-3} - \left[\frac{x}{2} e^{2x-3} - \int \frac{1}{2} e^{2x-3} dx \right] \\ &= \frac{x^2}{2} e^{2x-3} - \frac{x}{2} e^{2x-3} + \frac{1}{4} e^{2x-3} + C = e^{2x-3} \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4} \right) + C \end{aligned}$$

$$(15) \quad u = (\ln(x))^2 \quad du = 2 \frac{\ln(x)}{x} dx$$

$$dv = dx \quad v = x$$

$$\int (\ln(x))^2 dx = x(\ln(x))^2 - 2 \int \ln x dx$$

نستخدم القاعدة مرّة ثانية لإيجاد $\int \ln x dx$

$$u = \ln x \quad du = \frac{dx}{x}$$

$$dv = dx \quad v = x$$

$$\int (\ln(x))^2 dx = x(\ln(x))^2 - 2 \left[x \ln x - \int dx \right] = x(\ln(x))^2 - 2x \ln x + 2x + C$$

$$(16) \quad u = \sin x \quad dv = e^{2x} dx$$

$$du = \cos x dx \quad v = \frac{1}{2} e^{2x}$$

$$\int e^{2x} \sin x dx = \frac{1}{2} \sin x e^{2x} - \frac{1}{2} \int \cos x e^{2x} dx$$

نستخدم القاعدة مرّة ثانية فنحصل على:

$$u = \cos x \quad dv = e^{2x} dx$$

$$du = -\sin x dx \quad v = \frac{1}{2} e^{2x}$$

$$\begin{aligned} \int e^{2x} \sin x dx &= \frac{1}{2} \sin x e^{2x} - \frac{1}{2} \left[\frac{1}{2} \cos x e^{2x} + \frac{1}{2} \int e^{2x} \sin x dx \right] \\ &= \frac{1}{2} \sin x e^{2x} - \frac{1}{4} \cos x e^{2x} - \frac{1}{4} \int e^{2x} \sin x dx \end{aligned}$$

$$\frac{5}{4} \int e^{2x} \sin x dx = \frac{1}{2} \sin x e^{2x} - \frac{1}{4} \cos x e^{2x} \implies \int e^{2x} \sin x dx = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + C$$

$$(17) \quad u = \sin(\ln x) \quad du = \frac{\cos(\ln x)}{x} dx$$

$$dv = dx \quad v = x$$

$$\int \sin(\ln x) dx = x \sin(\ln x) - \int \cos(\ln x) dx$$

نستخدم القاعدة مرّة ثانية لإيجاد $\int \cos(\ln x) dx$

$$u = \cos(\ln x) \quad du = -\frac{\sin(\ln x)}{x} dx$$

$$dv = dx \quad v = x$$

$$\int \sin(\ln x) dx = x \sin(\ln x) - [x \cos(\ln x)] + \int x \cdot \frac{\sin(\ln x)}{x} dx = x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x) dx$$

$$\implies 2 \int \sin(\ln x) dx = x \sin(\ln x) - x \cos(\ln x)$$

$$\int \sin(\ln x) dx = \frac{x}{2} [\sin(\ln x) - \cos(\ln x)] + C$$

المجموعة B تمارين موضوعية

- (1) (a) (2) (a) (3) (a) (4) (b)
 (5) (a) (6) (b) (7) (b) (8) (d)
 (9) (b) (10) (b) (11) (c)

تمرن 5-6

التكامل باستخدام الكسور الجزئية

المجموعة A تمارين مقالية

$$(1) f(x) = \frac{A_1}{x-5} + \frac{A_2}{x-3}$$

$$2 = A_1(x-3) + A_2(x-5)$$

$$A_2 = -1 \quad \therefore \text{عوض عن } x \text{ بـ } 3$$

$$A_1 = 1 \quad \therefore \text{عوض عن } x \text{ بـ } 5$$

$$\therefore f(x) = \frac{1}{x-5} - \frac{1}{x-3}$$

$$\int f(x) dx = \ln|x-5| - \ln|x-3| + C$$

$$(2) x^2 + 2x = x(x+2)$$

$$f(x) = \frac{A_1}{x} + \frac{A_2}{(x+2)}$$

$$1 = A_1(x+2) + A_2x$$

$$A_2 = -\frac{1}{2} \quad \therefore \text{عوض عن } x \text{ بـ } -2$$

$$A_1 = \frac{1}{2} \quad \therefore \text{عوض عن } x \text{ بـ } 0$$

$$\therefore f(x) = \frac{1}{2x} - \frac{1}{2(x+2)}$$

$$\int f(x) dx = \frac{1}{2} \ln|x| - \frac{1}{2} \ln|x+2| + C$$

$$(3) x^2 + x + 12 = (x-3)(x+4)$$

$$f(x) = \frac{A_1}{x-3} + \frac{A_2}{x+4}$$

$$-x + 10 = A_1(x+4) + A_2(x-3)$$

$$A_2 = -2 \quad \therefore \text{عوض عن } x \text{ بـ } -4$$

$$A_1 = 1 \quad \therefore \text{عوض عن } x \text{ بـ } 3$$

$$\therefore f(x) = \frac{1}{x-3} - \frac{2}{x+4}$$

$$\int f(x)dx = \ln|x-3| - 2\ln|x+4| + C$$

$$(4) f(x) = \frac{A_1}{x} + \frac{A_2}{x-1} + \frac{A_3}{x+3}$$

$$12 = A_1(x-1)(x+3) + A_2(x)(x+3) + A_3(x)(x-1)$$

$$A_1 = -4 \quad \therefore \quad 0 \text{ بـ } x \text{ عوّض}$$

$$A_2 = 3 \quad \therefore \quad 1 \text{ بـ } x \text{ عوّض}$$

$$A_3 = 1 \quad \therefore \quad -3 \text{ بـ } x \text{ عوّض}$$

$$f(x) = \frac{-4}{x} + \frac{3}{x-1} + \frac{1}{x+3}$$

$$\int f(x)dx = -4\ln|x| + 3\ln|x-1| + \ln|x+3| + C$$

$$(5) 2x^2 + 5x - 3 = (2x-1)(x+3)$$

$$\frac{x+17}{(2x-1)(x+3)} = \frac{A_1}{2x-1} + \frac{A_2}{x+3}$$

$$x+17 = A_1(x+3) + A_2(2x-1)$$

$$A_1 = 5 \quad \therefore \quad \frac{1}{2} \text{ بـ } x \text{ عوّض}$$

$$A_2 = -2 \quad \therefore \quad -3 \text{ بـ } x \text{ عوّض}$$

$$\int \frac{x+17}{2x^2+5x-3} dx = \int \left(\frac{5}{2x-1} - \frac{2}{x+3} \right) dx$$

$$= \frac{5}{2} \ln|2x-1| - 2\ln|x+3| + C$$

$$(6) x^3 - 6x^2 + 9x = x(x^2 - 6x + 9) = x(x-3)^2$$

$$\frac{-6x+25}{x^3-6x^2+9x} = \frac{A_1}{x} + \frac{A_2}{(x-3)} + \frac{A_3}{(x-3)^2}$$

$$\therefore -6x+25 = A_1(x-3)^2 + A_2x(x-3) + A_3x$$

$$A_3 = \frac{7}{3} \quad \therefore \quad 3 \text{ بـ } x \text{ عوّض}$$

$$A_1 = \frac{25}{9} \quad \therefore \quad 0 \text{ بـ } x \text{ عوّض}$$

عوّض في المعادلة عن $A_1 = \frac{25}{9}$ و $A_3 = \frac{7}{3}$ ولتكن $x = 1$ لإيجاد قيمة A_2 .

$$\therefore A_2 = -\frac{25}{9}$$

$$\frac{-6x+25}{x^3-6x^2+9x} = \frac{25}{9x} - \frac{25}{9(x-3)} + \frac{7}{3(x-3)^2}$$

$$\int \frac{-6x+25}{x^3-6x^2+9x} dx = \frac{25}{9} \ln|x| - \frac{25}{9} \ln|x-3| - \frac{7}{3} \times \frac{1}{(x-3)} + C$$

$$(7) \quad x^3 - 3x^2 = x^2(x-3)$$

$$\frac{3x^2 - 4x + 3}{x^3 - 3x^2} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x-3}$$

$$\therefore 3x^2 - 4x + 3 = A_1x(x-3) + A_2(x-3) + A_3x^2$$

$$A_2 = -1 \quad \therefore \quad 0 \text{ بـ } x \text{ عوّض}$$

$$A_3 = 2 \quad \therefore \quad 3 \text{ بـ } x \text{ عوّض}$$

عوّض في المعادلة عن $A_2 = -1$ و $A_3 = 2$ ولتكن $x = 1$ لإيجاد قيمة A_1 .

$$\therefore A_1 = 1$$

$$\frac{3x^2 - 4x + 3}{x^3 - 3x^2} = \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x-3}$$

$$\int \frac{3x^2 - 4x + 3}{x^3 - 3x^2} dx = \ln|x| + \frac{1}{x} + 2 \ln|x-3| + C$$

$$(8) \quad \frac{x^2 + 3x + 2}{(x-3)^2} = 1 + \frac{9x-7}{(x-3)^2}$$

$$\frac{9x-7}{(x-3)^2} = \frac{A_1}{x-3} + \frac{A_2}{(x-3)^2}$$

$$9x-7 = A_1(x-3) + A_2$$

$$A_2 = 20 \quad \therefore \quad 3 \text{ بـ } x \text{ عوّض}$$

عوّض عن A_2 بـ 20 ولتكن $x = 1$ لإيجاد قيمة A_1 .

$$\therefore A_1 = 9$$

$$\frac{x^2 + 3x + 2}{(x-3)^2} = 1 + \frac{9}{x-3} + \frac{20}{(x-3)^2}$$

$$\int \frac{x^2 + 3x + 2}{(x-3)^2} dx = x + 9 \ln|x-3| - \frac{20}{x-3} + C$$

$$(9) \quad \frac{2x^2 + x + 3}{x^2 - 1} = 2 + \frac{x+5}{x^2 - 1}$$

$$\frac{x+5}{x^2 - 1} = \frac{A_1}{x-1} + \frac{A_2}{x+1}$$

$$x+5 = A_1(x+1) + A_2(x-1)$$

$$A_1 = 3 \quad \therefore \quad 1 \text{ بـ } x \text{ عوّض}$$

$$A_2 = -2 \quad \therefore \quad -1 \text{ بـ } x \text{ عوّض}$$

$$\frac{2x^2 + x + 3}{x^2 - 1} = 2 + \frac{3}{x-1} - \frac{2}{x+1}$$

$$\int \frac{2x^2 + x + 3}{x^2 - 1} dx = \int \left(2 + \frac{3}{x-1} - \frac{2}{x+1} \right) dx$$

$$= 2x + 3 \ln|x-1| - 2 \ln|x+1| + C$$

$$(10) \frac{x^3 - 2}{x^2 + x} = x - 1 + \frac{x - 2}{x^2 + x}$$

$$\frac{x - 2}{x^2 + x} = \frac{A_1}{x} + \frac{A_2}{x + 1}$$

$$x - 2 = A_1(x + 1) + A_2x$$

$A_1 = -2 \quad \therefore \quad 0 \rightarrow x$ عوّض عن

$A_2 = 3 \quad \therefore \quad -1 \rightarrow x$ عوّض عن

$$\frac{x^3 - 2}{x^2 + x} = x - 1 - \frac{2}{x} + \frac{3}{x + 1}$$

$$\int \frac{x^3 - 2}{x^2 + x} dx = \frac{x^2}{2} - x - 2 \ln|x| + 3 \ln|x + 1| + C$$

$$(11) \frac{x^4 - 2x^3 + x^2 + 2x - 1}{x^2 - 2x + 1} = x^2 + \frac{2x - 1}{x^2 - 2x + 1}$$

$$\frac{2x - 1}{(x - 1)^2} = \frac{A_1}{x - 1} + \frac{A_2}{(x - 1)^2}$$

$$2x - 1 = A_1(x - 1) + A_2$$

$A_2 = 1 \quad \therefore \quad 1 \rightarrow x$ عوّض عن

$A_1 = 2 \quad \therefore \quad x = 0$ ولتكن A_2 بـ 1 ولتكن

$$\frac{x^4 - 2x^3 + x^2 + 2x - 1}{x^2 - 2x + 1} = x^2 + \frac{2}{x - 1} + \frac{1}{(x - 1)^2}$$

$$\int \frac{x^4 - 2x^3 + x^2 + 2x - 1}{x^2 - 2x + 1} dx = \frac{x^3}{3} + 2 \ln|x - 1| - \frac{1}{(x - 1)} + C$$

$$(12) (a) f(x) = \frac{(x - 2)(2x^3 - x^2 - 9x + 14)}{(x - 2)^2(x + 2)} = \frac{2x^3 - x^2 - 9x + 14}{x^2 - 4}$$

$$= 2x - 1 + \frac{-x + 10}{(x - 2)(x + 2)}$$

$$(b) \frac{-x + 10}{(x - 2)(x + 2)} = \frac{A_1}{x - 2} + \frac{A_2}{x + 2} = \frac{2}{x - 2} - \frac{3}{x + 2}$$

$$(c) f(x) = 2x - 1 + \frac{2}{x - 2} + \frac{-3}{x + 2}$$

$$\int f(x) dx = x^2 - x + 2 \ln|x - 2| - 3 \ln|x + 2| + C$$

المجموعة B تمارين موضوعية

- | | | | |
|---------|----------|---------|---------|
| (1) (b) | (2) (b) | (3) (a) | (4) (a) |
| (5) (d) | (6) (c) | (7) (b) | (8) (c) |
| (9) (c) | (10) (d) | | |

المجموعة A تمارين مقالية

$$(1) \int_{-1}^1 (3x^2 - 12x) dx = [x^3 - 6x^2]_{-1}^1 = 2$$

$$(2) \int_0^2 (x^2 + 2x + 1) dx = \left[\frac{x^3}{3} + x^2 + x \right]_0^2 = \frac{26}{3}$$

$$(3) \int_0^4 \frac{(x-1)(x+1)}{(x+1)} dx = \int_0^4 (x-1) dx = \left[\frac{x^2}{2} - x \right]_0^4 = 4$$

$$(4) \int_0^{\frac{\pi}{3}} \cos 3x dx = \frac{1}{3} \sin 3x \Big|_0^{\frac{\pi}{3}} = \frac{1}{3} [\sin \pi - \sin 0] = 0$$

$$(5) \int_1^4 \left(\frac{4}{x^2} - \frac{x^2}{2} \right) dx = \left(-\frac{4}{x} \right) \Big|_1^4 - \left(\frac{x^3}{6} \right) \Big|_1^4 = -\frac{15}{2}$$

$$(6) \int_0^1 x \cdot x^{\frac{1}{2}} dx = \int_0^1 x^{\frac{3}{2}} dx = \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^1 = \frac{2}{5}$$

$$(7) [3e^x + 5 \ln|x|]_1^2 = 3(e^2 - e) + 5 \ln 2$$

$$(8) \int_{-1}^2 (-x+2) dx + \int_2^3 (x-2) dx = \left[-\frac{x^2}{2} + 2x \right]_{-1}^2 + \left[\frac{x^2}{2} - 2x \right]_2^3 = 5$$

$$(9) \int_{-1}^0 -x^3 dx + \int_0^1 x^3 dx = -\left[\frac{x^4}{4} \right]_{-1}^0 + \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{2}$$

$$(10) \int_{-2}^0 (-x^2 + 3) dx + \int_0^3 (x^2 + 3) dx = \left[-\frac{x^3}{3} + 3x \right]_{-2}^0 + \left[\frac{x^3}{3} + 3x \right]_0^3 = \frac{64}{3}$$

$$(11) x^2 + 2x - 8 = (x-2)(x+4)$$

$$x_1 = -4, \quad x_2 = 2$$

x		-4		2	
$x^2 + 2x - 8$	$+$	0	$-$	0	$+$

$$\therefore x^2 + 2x - 8 \leq 0 \quad \therefore \forall x \in [-4, 2]$$

$$\int_{-4}^2 (x^2 + 2x - 8) dx \leq 0$$

$$(12) x^3 - 5x^2 - 6x = x(x^2 - 5x - 6) = x(x+1)(x-6)$$

\longleftarrow	$-$	$+$	$-$	$+$	\longrightarrow
	-1	0	6		

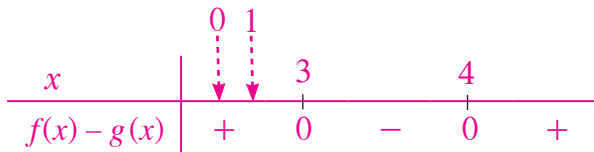
$$x^3 - 5x^2 - 6x \geq 0 \quad \forall x \in [-1, 0]$$

$$\int_{-1}^0 (x^3 - 5x^2 - 6x) dx \geq 0$$

$$(13) f(x) = x^2 - 3x + 7$$

$$g(x) = 4x - 5$$

$$f(x) - g(x) = x^2 - 7x + 12 = (x - 3)(x - 4)$$



$$f(x) - g(x) \geq 0 \quad \forall x \in [0, 1]$$

$$\int_0^1 (f(x) - g(x)) dx \geq 0 \implies \int_0^1 f(x) dx \geq \int_0^1 g(x) dx$$

$$\int_0^1 (x^2 - 3x + 7) dx \geq \int_0^1 (4x - 5) dx$$

$$(14) y = \sqrt{9 - x^2} \quad \therefore y^2 = 9 - x^2 \quad \therefore y^2 + x^2 = 9$$

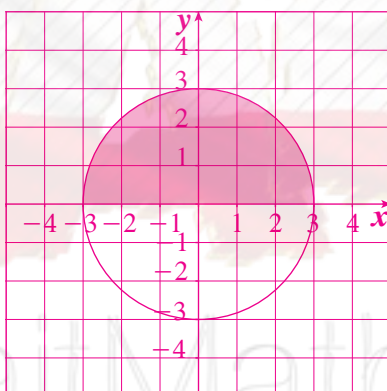
وهي معادلة دائرة مركزها نقطة الأصل ونصف قطرها 3 وحدات.

والدالة $y = \sqrt{9 - x^2}$ تمثل معادلة النصف العلوي للدائرة.

$$\therefore \int_{-3}^3 \sqrt{9 - x^2} dx$$

$$= \frac{1}{2} \pi (3)^2 = \frac{9}{2} \pi$$

\therefore مساحة المنطقة المظللة تساوي:



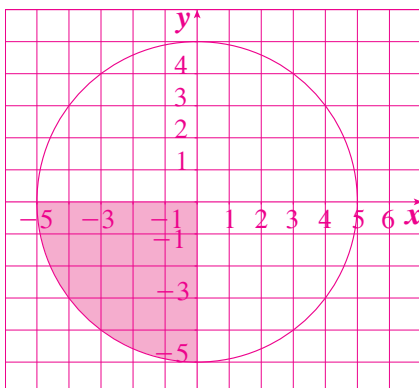
$$(15) y = -\sqrt{25 - x^2} \quad \therefore y^2 = 25 - x^2 \quad \therefore y^2 + x^2 = 25$$

وهي معادلة دائرة مركزها نقطة الأصل ونصف قطرها 5 وحدات.

والدالة $y = -\sqrt{25 - x^2}$ تمثل معادلة النصف السفلي للدائرة.

$$\int_{-5}^0 -\sqrt{25 - x^2} dx = -A$$

$$= -\frac{1}{4} \pi (5)^2 = -\frac{25}{4} \pi$$



(16) $u = 1 + x$, $du = dx$

$$\int_0^3 \frac{dx}{(1+x)^2} = \int_1^4 \frac{1}{u^2} du = -\left[\frac{1}{u}\right]_1^4 = \frac{3}{4}$$

(17) $u = \ln x$, $du = \frac{dx}{x}$

$$x = e$$
 , $u = 1$

$$x = 6$$
 , $u = \ln 6$

$$\int_1^{\ln 6} \frac{du}{u} = [\ln|u|]_1^{\ln 6} = \ln(\ln 6)$$

(18) $u = \ln x$, $du = \frac{dx}{x}$

$$\int_1^e \frac{(\ln x)^6}{x} dx = \int_0^1 u^6 du = \left[\frac{u^7}{7}\right]_0^1 = \frac{1}{7}$$

(19) $u = x^2 + 1$, $du = 2x dx$

$$x = -1 \implies u = 2$$
 , $x = 3 \implies u = 10$

$$\int_{-1}^3 \frac{x dx}{x^2 + 1} = \frac{1}{2} \int_2^{10} \frac{du}{u} = \frac{1}{2} [\ln|u|]_2^{10} = \frac{1}{2} \ln 5$$

(20) $u = x$, $du = dx$

$$dv = \sin x dx$$
 , $v = -\cos x$

$$\int_0^{\frac{\pi}{2}} x \sin x dx = -x \cos x \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx = [\sin x]_0^{\frac{\pi}{2}} = 1$$

(21) $u = x$, $du = dx$

$$dv = \cos 3x dx$$
 , $v = \frac{1}{3} \sin 3x$

$$\int_0^{\pi} x \cos 3x dx = \frac{x}{3} \sin 3x \Big|_0^{\pi} - \int_0^{\pi} \frac{1}{3} \sin 3x dx = \left[\frac{1}{9} \cos 3x\right]_0^{\pi} = -\frac{2}{9}$$

(22) $u = \ln x$, $du = \frac{dx}{x}$

$$dv = x^3$$
 , $v = \frac{x^4}{4}$

$$\int_1^3 x^3 \ln x dx = \frac{x^4}{4} \ln x \Big|_1^3 - \int_1^3 \frac{x^3}{4} dx = \frac{81}{4} \ln 3 - \left[\frac{x^4}{16}\right]_1^3 = \frac{81}{4} \ln 3 - 5$$

(23) $\int_0^{\frac{\pi}{2}} e^{2x} \cos x dx$

$$u = \cos x \quad dv = e^{2x} dx$$

$$du = -\sin x dx \quad v = \frac{1}{2} e^{2x}$$

$$\int_0^{\frac{\pi}{2}} e^{2x} \cos x dx = \frac{1}{2} [e^{2x} \cos x]_0^{\frac{\pi}{2}} + \frac{1}{2} \int_0^{\frac{\pi}{2}} e^{2x} \sin x dx$$

$$= -\frac{1}{2} + \frac{1}{2} \int_0^{\frac{\pi}{2}} e^{2x} \sin x dx$$

$$u = \sin x \quad dv = e^{2x} dx$$

نطبق القاعدة مرّة ثانية على التكامل المحدد:

فيكون:

$$du = \cos x \, dx \quad v = \frac{1}{2}e^{2x}$$

$$\int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx = -\frac{1}{2} + \frac{1}{2} \left[\left(\frac{1}{2} e^{2x} \sin x \right)_0^{\frac{\pi}{2}} - \frac{1}{2} \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx \right]$$

$$\int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx = -\frac{1}{2} + \frac{1}{4} e^{\pi} - \frac{1}{4} \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx \implies \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx = \frac{e^{\pi}}{5} - \frac{2}{5}$$

$$(24) \quad \frac{4}{x^2 - 4} = \frac{A_1}{(x-2)} + \frac{A_2}{(x+2)}$$

$$4 = A_1(x+2) + A_2(x-2)$$

$$A_2 = -1 \quad \therefore \quad -2 \text{ بـ } x$$

$$A_1 = 1 \quad \therefore \quad 2 \text{ بـ } x$$

$$\frac{4}{x^2 - 4} = \frac{1}{x-2} - \frac{1}{x+2}$$

$$\int_{-1}^1 \frac{4}{x^2 - 4} \, dx = [\ln|x-2| - \ln|x+2|]_{-1}^1 = -2 \ln 3$$

$$(25) \quad x^2 + 2x - 3 = (x-1)(x+3)$$

$$\frac{5x-1}{x^2 + 2x - 3} = \frac{A_1}{x-1} + \frac{A_2}{x+3}$$

$$5x - 1 = A_1(x+3) + A_2(x-1)$$

$$A_2 = 4 \quad \therefore \quad -3 \text{ بـ } x$$

$$A_1 = 1 \quad \therefore \quad 1 \text{ بـ } x$$

$$\frac{5x-1}{x^2 + 2x - 3} = \frac{1}{x-1} + \frac{4}{x+3}$$

$$\int_{-2}^0 \frac{5x-1}{x^2 + 2x - 3} \, dx = [\ln|x-1| + 4 \ln|x+3|]_{-2}^0 = 3 \ln 3$$

$$(26) \quad \frac{x^2 + 2x + 1}{(x+1)^2} = \frac{x^2}{(x+1)^2} + \frac{x^2 + 2x + 1}{(x+1)^2}$$

$$\frac{x^2}{(x+1)^2} = 1 + \frac{-2x-1}{(x+1)^2}$$

$$\frac{-2x-1}{(x+1)^2} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2}$$

$$-2x - 1 = A_1(x+1) + A_2$$

$$A_2 = +1, \quad -1 \text{ بـ } x$$

$$A_1 = -2 \text{ نجد } A_2 \text{ مع قيمة } 0 \text{ بـ } x$$

$$\frac{x^2}{(x+1)^2} = 1 + \frac{-2}{x+1} + \frac{1}{(x+1)^2}$$

$$\int_1^3 \frac{x^2}{(x+1)^2} \, dx = \int_1^3 \left[1 - \frac{2}{x+1} + \frac{1}{(x+1)^2} \right] \, dx = \left[x - 2 \ln|x+1| + \frac{-1}{x+1} \right]_1^3 = \frac{9}{4} - 2 \ln 2$$

$$\int_1^3 \frac{x^2}{(x+1)^2} dx$$

$$u = x + 1 \implies du = dx$$

$$x = u - 1$$

$$\begin{aligned} \int_1^3 \frac{x^2}{(x+1)^2} dx &= \int_2^4 \frac{(u-1)^2}{u^2} du \\ &= \int_2^4 \frac{(u^2 - 2u + 1)}{u^2} du \\ &= \int_2^4 \left(1 - \frac{2}{u} + \frac{1}{u^2}\right) du \\ &= \left[u - 2\ln|u| - \frac{1}{u}\right]_2^4 \\ &= \left(4 - 2\ln 4 - \frac{1}{4}\right) - \left(2 - 2\ln 2 - \frac{1}{2}\right) \\ &= \frac{9}{4} - 2\ln 2 \end{aligned}$$

المجموعة B تمارين موضوعية

- | | | | |
|----------|----------|----------|----------|
| (1) (a) | (2) (b) | (3) (b) | (4) (a) |
| (5) (b) | (6) (b) | (7) (b) | (8) (c) |
| (9) (c) | (10) (a) | (11) (b) | (12) (d) |
| (13) (d) | (14) (b) | (15) (c) | |

اختبار الوحدة الخامسة

$$(1) F'(x) = \frac{1}{3} \cdot \frac{3}{2} (2x^2 + 6x + 5)^{\frac{1}{2}} (4x + 6) = (2x + 3) \sqrt{2x^2 + 6x + 5} = f(x)$$

$$(2) F(x) = x^3 - x^2 + C$$

$$F(2) = 6 \quad ; \quad C = 2$$

$$F(x) = x^3 - x^2 + 2$$

$$(3) \frac{1}{2} \int (2x+4)(x^2+4x+7)^{\frac{1}{2}} dx = \frac{(x^2+4x+7)^{\frac{3}{2}}}{3} + C$$

$$(4) \int (2x-1)(x^2-x+7)^{-5} dx = \frac{(x^2-x+7)^{-4}}{-4} + C = \frac{-1}{4(x^2-x+7)^4} + C$$

$$(5) u = x - 3 \quad , \quad x^2 = (u+3)^2 \quad , \quad du = dx$$

$$\begin{aligned} \int x^2 \sqrt[3]{x-3} dx &= \int (u+3)^2 \cdot u^{\frac{1}{3}} du = \int (u^{\frac{5}{3}} + 6u^{\frac{4}{3}} + 9u^{\frac{1}{3}}) du \\ &= \frac{3u^{\frac{8}{3}}}{8} + \frac{18u^{\frac{7}{3}}}{7} + \frac{27}{4} u^{\frac{4}{3}} + C = \frac{3(x-3)^{\frac{8}{3}}}{8} + \frac{18(x-3)^{\frac{7}{3}}}{7} + \frac{27}{4}(x-3)^{\frac{4}{3}} + C \end{aligned}$$

$$(6) \quad u = x^2 - 8, \quad x^2 = u + 8, \quad du = 2x dx \implies x dx = \frac{1}{2} du$$

$$\begin{aligned} \int x^3 \sqrt{x^2 - 8} dx &= \int x^2 \sqrt{x^2 - 8} (x dx) \\ &= \frac{1}{2} \int (u + 8) u^{\frac{1}{2}} du = \frac{1}{2} \int (u^{\frac{3}{2}} + 8u^{\frac{1}{2}}) du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{8u^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{(x^2 - 8)^{\frac{5}{2}}}{5} + \frac{8(x^2 - 8)^{\frac{3}{2}}}{3} + C \end{aligned}$$

$$(7) \quad \int \frac{x+1}{\sqrt[3]{x+1}} dx = \int \frac{(\sqrt[3]{x+1} + 1)(\sqrt[3]{x^2 - \sqrt[3]{x+1}})}{(\sqrt[3]{x+1})} dx$$

$$= \int (x^{\frac{2}{3}} - x^{\frac{1}{3}} + x) dx = \frac{3}{5} x^{\frac{5}{3}} - \frac{3}{4} x^{\frac{4}{3}} + x + C$$

$$(8) \quad \int \cos x (\sin x)^{-3} dx = \frac{(\sin x)^{-2}}{-2} + C = -\frac{1}{2 \sin^2 x} + C$$

$$(9) \quad -\int -\sin x (\cos x)^{\frac{2}{3}} dx = -\frac{3 \cos x^{\frac{5}{3}}}{5} + C$$

$$(10) \quad \int \sec^7 x \tan x dx = \int \sec^6 x (\tan x \cdot \sec x dx)$$

$$u = \sec x, \quad du = \tan x \sec x dx$$

$$\int \sec^6 x (\tan x \sec x dx) = \int u^6 du = \frac{\sec^7 x}{7} + C$$

$$(11) \quad \frac{1}{3} \int 3e^{3x} dx + \frac{1}{2} \int \frac{2}{2x-1} dx = \frac{1}{3} e^{3x} + \frac{1}{2} \ln|2x-1| + C$$

$$(12) \quad 2 \int \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx = 2 e^{\sqrt{x}} + C$$

$$(13) \quad \frac{1}{3} \int \frac{3x^2 - 12}{x^3 - 6x^2 + 1} dx = \frac{1}{3} \ln|x^3 - 6x^2 + 1| + C$$

$$(14) \quad \frac{1}{2} \int \frac{2e^{2x} + 2x}{e^{2x} + x^2 + 3} dx = \frac{1}{2} \ln|e^{2x} + x^2 + 3| + C$$

$$(15) \quad \int (x^2 - 4) \cos x dx = \int x^2 \cos x dx - 4 \int \cos x dx = \int x^2 \cos x dx - 4 \sin x + C_1$$

في التكامل: $\int x^2 \cos x dx$

نأخذ:

$$u = x^2 \quad dv = \cos x dx$$

$$du = 2x dx \quad v = \sin x$$

$$\int x^2 \cos x dx = x^2 \sin x - 2 \int x \sin x dx$$

نستخدم القاعدة مرة ثانية لنجد $\int x \sin x dx$

$$\int x^2 \cos x dx = x^2 \sin x + 2x \cos x - 2 \sin x + C_2$$

$$\int (x^2 - 4) \cos x dx = x^2 \sin x + 2x \cos x - 6 \sin x + C$$

$$(16) \quad u = \ln(3x+2) \implies du = \frac{3}{3x+2} dx$$

$$dv = dx \implies v = x$$

$$\begin{aligned} \int \ln(3x+2) dx &= x \ln(3x+2) - \int \frac{3x}{3x+2} dx = x \ln(3x+2) - \int \frac{3x+2-2}{3x+2} dx \\ &= x \ln(3x+2) - x + \frac{2}{3} \ln|3x+2| + C \end{aligned}$$

$$(17) \quad u = 3x \quad dv = e^{2x+1} dx$$

$$du = 3dx \quad v = \frac{1}{2} e^{2x+1}$$

$$\int 3x e^{2x+1} dx = \frac{1}{2} \cdot 3x e^{2x+1} - \frac{3}{2} \int e^{2x+1} dx$$

$$\int 3x e^{2x+1} dx = \frac{3}{2} x e^{2x+1} - \frac{3}{4} e^{2x+1} + C$$

$$\int 3x e^{2x+1} dx = \left(\frac{3}{2} x - \frac{3}{4} \right) e^{2x+1} + C$$

$$(18) \quad u = x^2 \quad du = 2x dx$$

$$dv = e^{2x-1} dx \quad v = \frac{1}{2} e^{2x-1}$$

$$\int x^2 e^{2x-1} dx = \frac{x^2}{2} e^{2x-1} - \int x e^{2x-1} dx$$

نستخدم القاعدة مرة ثانية

$$u = x \quad du = dx$$

$$dv = e^{2x-1} dx \quad v = \frac{1}{2} e^{2x-1}$$

$$\int x^2 e^{2x-1} dx = \frac{x^2}{2} e^{2x-1} - \frac{x}{2} e^{2x-1} + \int \frac{1}{2} e^{2x-1} dx = e^{2x-1} \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4} \right) + C$$

$$(19) \quad \frac{x^2 - 3x - 28 + 28}{x^2 - 3x - 28} = 1 + \frac{28}{x^2 - 3x - 28}$$

$$x^2 - 3x - 28 = (x-7)(x+4)$$

$$\frac{28}{x^2 - 3x - 28} = \frac{A_1}{x-7} + \frac{A_2}{x+4}$$

$$28 = A_1(x+4) + A_2(x-7)$$

$$A_2 = -\frac{28}{11} \quad \therefore \text{عوّض عن } x \text{ بـ } -4$$

$$A_1 = \frac{28}{11} \quad \therefore \text{عوّض عن } x \text{ بـ } 7$$

$$\frac{x^2 - 3x}{x^2 - 3x - 28} = 1 + \frac{28}{11(x-7)} - \frac{28}{11(x+4)}$$

$$\int \frac{x^2 - 3x}{x^2 - 3x - 28} dx = x + \frac{28}{11} \ln|x-7| - \frac{28}{11} \ln|x+4| + C$$

$$(20) \frac{x^4 + 2x^2 + 6x}{x^3 + 4x^2 + 4x} = \frac{x(x^3 + 2x + 6)}{x(x^2 + 4x + 4)} = \frac{x^3 + 2x + 6}{x^2 + 4x + 4}$$

$$\frac{x^3 + 2x + 6}{x^2 + 4x + 4} = x - 4 + \frac{14x + 22}{(x + 2)^2}$$

(باستخدام القسمة المطولة)

$$\frac{14x + 22}{(x + 2)^2} = \frac{A_1}{x + 2} + \frac{A_2}{(x + 2)^2}$$

$$14x + 22 = A_1(x + 2) + A_2$$

عوّض عن x بـ -2 نحصل على $A_2 = -6$

نضع $A_2 = -6$ ونأخذ $x = 0$ نحصل على $A_1 = 14$

$$\int \frac{x^4 + 2x^2 + 6x}{x^3 + 4x^2 + 4x} dx = \frac{x^2}{2} - 4x + 14 \ln|x + 2| + \frac{6}{x + 2} + C$$

$$(21) [\ln|x|]_1^e = \ln e - \ln 1 = 1$$

$$(22) -\int_{-1}^1 -2x \sin(1 - x^2) dx = [\cos(1 - x^2)]_{-1}^1 = 0$$

$$(23) \int_0^{\frac{5}{2}} (-2x + 5) dx + \int_{\frac{5}{2}}^5 (2x - 5) dx = [-x^2 + 5x]_0^{\frac{5}{2}} + [x^2 - 5x]_{\frac{5}{2}}^5 = \frac{25}{2}$$

$$(24) y = -\sqrt{36 - x^2} \quad \therefore y^2 = 36 - x^2 \quad \therefore y^2 + x^2 = 36$$

وهي معادلة دائرة مركزها نقطة الأصل ونصف قطرها 6 وحدات.

والدالة $y = -\sqrt{36 - x^2}$ تمثل النصف السفلي للدائرة.

$$\int_{-6}^0 -\sqrt{36 - x^2} dx \quad \therefore \text{مساحة المنطقة المظللة تساوي:}$$

$$= \frac{1}{4} \pi (6)^2 = 9\pi \text{ units}^2$$

$$(25) \frac{x^2 - 3}{x^2 - 3x + 2} = 1 + \frac{3x - 5}{x^2 - 3x + 2} \quad (\text{باستخدام القسمة المطولة})$$

$$\frac{3x - 5}{x^2 - 3x + 2} = \frac{A_1}{x - 2} + \frac{A_2}{x - 1}$$

$$3x - 5 = A_1(x - 1) + A_2(x - 2)$$

عوّض عن x بـ 1 $\therefore A_2 = 2$

عوّض عن x بـ 2 $\therefore A_1 = 1$

$$\frac{x^2 - 3}{x^2 - 3x + 2} = 1 + \frac{1}{x - 2} + \frac{2}{x - 1}$$

$$\int \frac{x^2 - 3}{x^2 - 3x + 2} dx = [x + \ln|x - 2| + 2 \ln|x - 1|]_3^5 = 2 + \ln 3 + 2 \ln 2$$

$$(26) \frac{x^3 - 2x^2 + 2}{x^3 + 6x^2 + 9x} = 1 + \frac{-8x^2 - 9x + 2}{x(x + 3)^2}$$

(باستخدام القسمة المطولة)

$$\frac{-8x^2 - 9x + 2}{x(x + 3)^2} = \frac{A_1}{x} + \frac{A_2}{x + 3} + \frac{A_3}{(x + 3)^2}$$

$$-8x^2 - 9x + 2 = A_1(x + 3)^2 + A_2x(x + 3) + A_3x$$

$$A_1 = \frac{2}{9} \quad \therefore \quad 0 \text{ بـ } x \text{ عوّض عن}$$

$$A_3 = \frac{43}{3} \quad \therefore \quad -3 \text{ بـ } x \text{ عوّض عن}$$

$$A_2 = -\frac{74}{9} \quad \therefore \quad x = 1 \text{ ولتكن } \frac{43}{3} \text{ بـ } A_3 \text{ وعن } \frac{2}{9} \text{ بـ } A_1 \text{ عوّض عن}$$

$$\frac{x^3 - 2x^2 + 2}{x^3 + 6x^2 + 9x} = 1 + \frac{2}{9x} - \frac{74}{9(x+3)} + \frac{43}{3(x+3)^2}$$

$$\int_1^3 \frac{x^3 - 2x^2 + 2}{x^3 + 6x^2 + 9x} dx = \left[x + \frac{2}{9} \ln|x| - \frac{74}{9} \ln|x+3| - \frac{43}{3(x+3)} \right]_1^3$$

$$= 2 + \frac{2}{9} \ln 3 + \frac{43}{36} - \frac{74}{9} (\ln 6 - \ln 4)$$

$$= \frac{115}{36} + \frac{2}{9} \ln 3 - \frac{74}{9} (\ln 6 - \ln 4)$$

$$(27) \quad -x^2 + 7x + 8 = (x+1)(-x+8)$$

$$\begin{array}{c|cccc} x & & -1 & & 8 \\ \hline -x^2 + 7x + 8 & - & 0 & + & 0 & - \end{array}$$

$$-x^2 + 7x + 8 \geq 0 \quad \forall x \in [2, 5]$$

$$\therefore \int_2^5 (-x^2 + 7x + 8) dx \geq 0$$

$$(28) \quad x^2 + 7x + 10 = (x+2)(x+5)$$

$$\begin{array}{c|ccc} x & & -5 & & -2 \\ \hline x^2 + 7x + 10 & + & 0 & - & 0 & + \end{array}$$

$$x^2 + 7x + 10 \leq 0 \quad \forall x \in [-4, -2]$$

$$\therefore \int_{-4}^{-2} (x^2 + 7x + 10) dx \leq 0$$

$$(29) \quad f(x) = x^2 + 13x + 9$$

$$g(x) = 5x - 6$$

$$f(x) - g(x) = x^2 + 8x + 15$$

$$f(x) - g(x) = (x+3)(x+5)$$

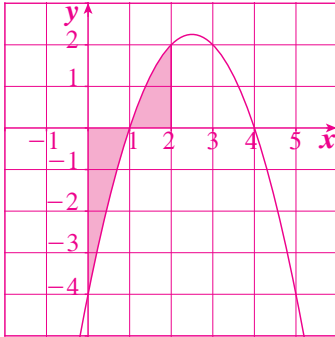
$$\begin{array}{c|ccc} & & -4 & \\ & & \downarrow & \\ x & & -5 & & -3 \\ \hline f(x) - g(x) & + & 0 & - & 0 & + \end{array}$$

$$f(x) - g(x) \leq 0 \quad \forall x \in [-5, -4]$$

$$\therefore \int_{-5}^{-4} (f(x) - g(x)) dx \leq 0 \implies \int_{-5}^{-4} f(x) dx \leq \int_{-5}^{-4} g(x) dx$$

$$\implies \int_{-5}^{-4} (x^2 + 13x + 9) dx \leq \int_{-5}^{-4} (5x - 6) dx$$

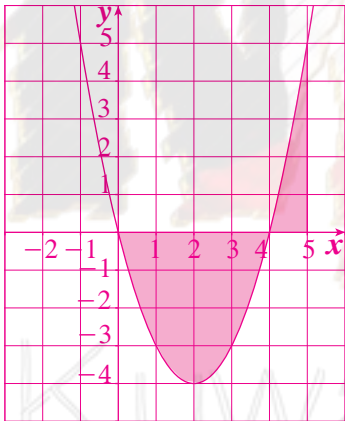
$$(1) (a) \int_0^2 (-x^2 + 5x - 4) dx = \left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right]_0^2 = -\frac{2}{3}$$



$$(b) A = \int_0^1 (x^2 - 5x + 4) dx + \int_1^2 (-x^2 + 5x - 4) dx$$

$$= \left[\frac{x^3}{3} - \frac{5}{2}x^2 + 4x \right]_0^1 + \left[-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x \right]_1^2 = 3 \text{ units square}$$

$$(2) (a) \int_0^5 (x^2 - 4x) dx = \left[\frac{x^3}{3} - 2x^2 \right]_0^5 = -\frac{25}{3}$$



$$(b) A = \int_0^4 (-x^2 + 4x) dx + \int_4^5 (x^2 - 4x) dx$$

$$= \left[-\frac{x^3}{3} + 2x^2 \right]_0^4 + \left[\frac{x^3}{3} - 2x^2 \right]_4^5 = 13 \text{ units square}$$

$$(3) u = \ln x \quad du = \frac{dx}{x}$$

$$dv = x^2 dx \quad v = \frac{x^3}{3}$$

$$\int x^2 \ln x dx = \left(\frac{x^3}{3} \right) \ln x - \int \frac{x^2}{3} dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$$

$$(4) \int \cos \theta \cdot (\sin \theta)^{-2} d\theta = -\frac{1}{\sin \theta} + C$$

$$(5) \frac{dy}{dx} = 2x - 3x^2 + C_1$$

$$y'(0) = 4 \quad \therefore C_1 = 4$$

$$\frac{dy}{dx} = 2x - 3x^2 + 4$$

$$y = x^2 - x^3 + 4x + C_2$$

$$y(0) = 1 \quad \therefore C_2 = 1$$

$$y = x^2 - x^3 + 4x + 1$$

$$(6) \quad C(x) = \int \frac{2}{\sqrt{x}} dx = 4\sqrt{x} + C_1$$

$$C(x) = \text{التكلفة}$$

$$x = \text{عدد النسخات}$$

$$\therefore C(25) = 50 \implies 50 = 4\sqrt{25} + C_1 \implies C_1 = 30$$

$$C(x) = 4\sqrt{x} + 30$$

$$C(2500) = 4\sqrt{2500} + 30 = 230$$

التكلفة: 230 دينارًا

$$(7) \quad u = x^3 \quad du = 3x^2 dx$$

$$dv = e^x dx \quad v = e^x$$

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

نستخدم القاعدة مرّة ثانية

$$u = x^2 \quad du = 2x dx$$

$$dv = e^x dx \quad v = e^x$$

$$\int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6 \int x e^x dx$$

نستخدم القاعدة مرّة ثالثة

$$u = x \quad du = dx$$

$$dv = e^x dx \quad v = e^x$$

$$\int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6x e^x - 6 \int e^x dx \\ = e^x (x^3 - 3x^2 + 6x - 6) + C$$

$$(8) \quad u = \ln x \quad du = \frac{dx}{x}$$

$$dv = x^3 dx \quad v = \frac{x^4}{4}$$

$$\int x^3 \ln x dx = \frac{x^4}{4} \cdot \ln x - \int \frac{x^3}{4} dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + C$$

$$(9) \quad (a) \quad 2x^2 - 5x + 3 = (2x - 3)(x - 1)$$

$$\frac{x - 2}{2x^2 - 5x + 3} = \frac{A_1}{2x - 3} + \frac{A_2}{x - 1}$$

$$x - 2 = A_1(x - 1) + A_2(2x - 3)$$

$$A_2 = 1 \quad \therefore 1 \text{ بـ } x$$

$$A_1 = -1 \quad \therefore \frac{3}{2} \text{ بـ } x$$

$$\frac{x-2}{2x^2-5x+3} = \frac{-1}{2x-3} + \frac{1}{x-1}$$

$$\int \frac{x-2}{2x^2-5x+3} dx = -\frac{1}{2} \ln|2x-3| + \ln|x-1| + C$$

(b) $x^2 + 10x + 25 = (x+5)^2$

$$\frac{x^2-9}{(2x+1)(x^2+10x+25)} = \frac{A_1}{2x+1} + \frac{A_2}{x+5} + \frac{A_3}{(x+5)^2}$$

$$x^2-9 = A_1(x+5)^2 + A_2(2x+1)(x+5) + A_3(2x+1)$$

$$A_3 = -\frac{16}{9} \quad \therefore \text{عوض عن } x \text{ بـ } -5$$

$$A_1 = -\frac{35}{81} \quad \therefore \text{عوض عن } x \text{ بـ } -\frac{1}{2}$$

$$A_2 = \frac{58}{81} \quad \therefore \text{عوض عن } A_3 \text{ بـ } -\frac{16}{9} \text{ و } A_2 \text{ بـ } -\frac{35}{81} \text{ ولتكن } x=0$$

$$\frac{x^2-9}{(2x+1)(x+5)^2} = \frac{-35}{81(2x+1)} + \frac{58}{81(x+5)} - \frac{16}{9(x+5)^2}$$

$$\int \frac{x^2-9}{(2x+1)(x+5)^2} dx = \frac{-35}{162} \ln|2x+1| + \frac{58}{81} \ln|x+5| + \frac{16}{9(x+5)} + C$$

(c) $\frac{x^4+3x^2-7}{(x-1)(x^2+5x-6)} = x-4 + \frac{30x^2-50x+17}{(x-1)^2(x+6)}$ (باستخدام القسمة المطولة)

$$\frac{30x^2-50x+17}{(x+6)(x-1)^2} = \frac{A_1}{x+6} + \frac{A_2}{x-1} + \frac{A_3}{(x-1)^2}$$

$$30x^2-50x+17 = A_1(x-1)^2 + A_2(x-1)(x+6) + A_3(x+6)$$

$$A_3 = -\frac{3}{7} \quad \therefore \text{عوض عن } x \text{ بـ } 1$$

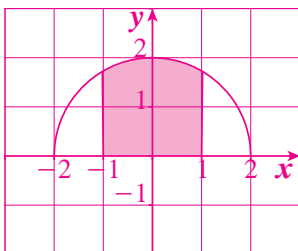
$$A_1 = \frac{1397}{49} \quad \therefore \text{عوض عن } x \text{ بـ } -6$$

$$A_2 = \frac{73}{49} \quad \therefore \text{عوض عن } A_1 \text{ بـ } \frac{1397}{49} \text{ و } A_2 \text{ بـ } -\frac{3}{7} \text{ ولتكن } x=0$$

$$\frac{x^4+3x^2-7}{(x-1)(x^2+5x-6)} = x-4 + \frac{1397}{49(x+6)} + \frac{73}{49(x-1)} - \frac{3}{7(x-1)^2}$$

$$\int \frac{x^4+3x^2-7}{(x-1)(x^2+5x-6)} dx = \frac{x^2}{2} - 4x + \frac{1397}{49} \ln|x+6| + \frac{73}{49} \ln|x-1| + \frac{3}{7(x-1)} + C$$

(10)



لايجاد التكامل المحدد: $\int_{-1}^1 \sqrt{4-x^2} dx$

نفترض: $x = 2 \cos \theta \implies dx = -2 \sin \theta d\theta$

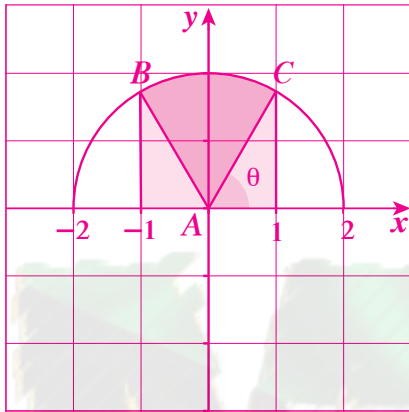
عند $x = -1$ تكون $\theta = \frac{2\pi}{3}$ ؛ عند $x = 1$ تكون $\theta = \frac{\pi}{3}$

$$\int_{-1}^1 \sqrt{4-x^2} dx = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (\sqrt{4-4\cos^2\theta})(2\sin\theta d\theta)$$

$$= \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 4\sin^2\theta d\theta = 2 \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (1-\cos 2\theta) d\theta = \frac{2\pi}{3} + \sqrt{3}$$

حل آخر

$$\cos\theta = \frac{1}{2} \implies \theta = \frac{\pi}{3}$$



قياس زاوية القطاع الدائري (ABC) هو أيضًا $\frac{\pi}{3}$

فتكون مساحته تساوي $\frac{1}{2} \times \frac{\pi}{3} (2)^2$

أي أن مساحة (ABC) $= \frac{2\pi}{3}$

مساحة كل مثلث $= \frac{1}{2} \times 1 \times \sqrt{3} = \frac{\sqrt{3}}{2}$

مساحة المثلثين $= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

المساحة الإجمالية $= \frac{2\pi}{3} + \sqrt{3}$

$$\int_{-1}^1 \sqrt{4-x^2} dx = \frac{2\pi}{3} + \sqrt{3}$$

$$(11) \int_{-4}^4 \frac{1}{\pi} \sqrt{16-x^2} dx - \int_{-4}^4 x \sqrt{16-x^2} dx$$

$$= \frac{1}{\pi} \int_{-4}^4 \sqrt{16-x^2} dx + \frac{1}{2} \int_{-4}^4 -2x \sqrt{16-x^2} dx$$

$$= \frac{1}{\pi} \left(\frac{1}{2} \right) (\pi) (4)^2 + \frac{1}{2} \times \frac{2}{3} [(16-x^2)^{\frac{3}{2}}]_{-4}^4 = 8 + \frac{1}{3} (0) = 8$$

$$(12) x^2 + 5x + 4 = (x+1)(x+4)$$

$$\frac{2x+3}{x^2+5x+4} = \frac{A_1}{x+1} + \frac{A_2}{x+4}$$

$$2x+3 = A_1(x+4) + A_2(x+1)$$

عوض عن $x = -1$ $\therefore A_1 = \frac{1}{3}$

عوض عن $x = -4$ $\therefore A_2 = \frac{5}{3}$

$$\frac{2x+3}{x^2+5x+4} = \frac{1}{3(x+1)} + \frac{5}{3(x+4)}$$

$$\int \frac{2x+3}{x^2+5x+4} dx = \left[\frac{1}{3} \ln|x+1| + \frac{5}{3} \ln|x+4| \right]_0^2 = \frac{1}{3} \ln 3 + \frac{5}{3} \ln 6 - \frac{5}{3} \ln 4 = 2 \ln 3 - \frac{5}{3} \ln 2$$

$$(13) \frac{x^3-6x^2+3}{x^3-6x^2+9x} = 1 + \frac{-9x+3}{x^3-6x^2+9x}$$

$$x^3-6x^2+9x = x(x-3)^2$$

$$\frac{-9x+3}{x(x-3)^2} = \frac{A_1}{x} + \frac{A_2}{x-3} + \frac{A_3}{(x-3)^2}$$

$$-9x+3 = A_1(x-3)^2 + A_2x(x-3) + A_3x$$

(باستخدام القسمة المطولة)

$$A_1 = \frac{1}{3} \quad \therefore \text{عوض عن } x \text{ بـ } 0$$

$$A_3 = -8 \quad \therefore \text{عوض عن } x \text{ بـ } 3$$

$$A_2 = -\frac{1}{3} \quad \therefore \text{عوض عن } A_1 \text{ بـ } \frac{1}{3} \text{ وعن } A_3 \text{ بـ } -8 \text{ ولتكن } x = 1$$

$$\frac{x^3 - 6x^2 + 3}{x^3 - 6x^2 + 9x} = 1 + \frac{1}{3x} - \frac{1}{3(x-3)} - \frac{8}{(x-3)^2}$$

$$\int_1^2 \frac{x^3 - 6x^2 + 3}{x^3 - 6x^2 + 9x} dx = \left[x + \frac{1}{3} \ln|x| - \frac{1}{3} \ln|x-3| + \frac{8}{x-3} \right]_1^2 = -3 + \frac{2}{3} \ln 2$$

$$(14) \int_3^5 x^3 \sqrt{x^2 - 4} dx = \int_3^5 x^2 \sqrt{x^2 - 4} (x dx)$$

$$u = x^2 - 4 \implies du = 2x dx$$

$$x^2 = u + 4$$

$$\frac{1}{2} \int_5^{21} (u+4)u^{\frac{1}{2}} du = \frac{1}{2} \int_5^{21} u^{\frac{3}{2}} du + 2 \int_5^{21} u^{\frac{1}{2}} du = \frac{1}{5} [u^{\frac{5}{2}}]_5^{21} + \frac{4}{3} [u^{\frac{3}{2}}]_5^{21} = \frac{581}{5} \sqrt{21} - \frac{35}{3} \sqrt{5}$$

$$(15) -x^2 + 9x - 18 = (-x+6)(x-3)$$

x				
$-x^2 + 9x - 18$	0	2	3	6
	-	0	+	0
	-	0	+	0

$$-x^2 + 9x - 18 \leq 0 \quad \forall x \in [0, 2]$$

$$\therefore \int_0^2 (-x^2 + 9x - 18) \leq 0$$

$$(16) f(x) - g(x) = x^2 + 13x + 15 - 3x + 6 = x^2 + 10x + 21$$

$$f(x) - g(x) = (x+3)(x+7)$$

x					
$f(x) - g(x)$	-7	-3	-1	2	
	+	0	-	0	+
	+	0	-	0	+

$$f(x) - g(x) \geq 0 \quad \forall x \in [-1, 2]$$

$$\int_{-1}^2 (f(x) - g(x)) dx \geq 0$$

$$\therefore \int_{-1}^2 f(x) dx \geq \int_{-1}^2 g(x) dx$$

$$\therefore \int_{-1}^2 (x^2 + 13x + 15) dx \geq \int_{-1}^2 (3x - 6) dx$$

$$\therefore \int_{-1}^2 f(x) dx \geq \int_{-1}^2 g(x) dx$$

$$\therefore \int_{-1}^2 (x^2 + 13x + 15) dx \geq \int_{-1}^2 (3x - 6) dx$$

المجموعة A تمارين مقالية

$$(1) A = \int_1^3 8x^3 dx = 2x^4 \Big|_1^3 = 160 \text{ units square}$$

$$(2) A = \int_0^5 (-x^2 + 5x) dx = \left[-\frac{x^3}{3} + \frac{5}{2}x^2 \right]_0^5 = \frac{125}{6} \text{ units square}$$

$$(3) A = \int_{-2\sqrt{3}}^{2\sqrt{3}} (12 - x^2) dx = \left[12x - \frac{x^3}{3} \right]_{-2\sqrt{3}}^{2\sqrt{3}} = 32\sqrt{3} \text{ units square}$$

$$(4) A = \int_{-3}^{-2} (x^2 - x - 6) dx + \int_{-2}^2 (-x^2 + x + 6) dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x \right]_{-3}^{-2} + \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x \right]_{-2}^2 = \frac{43}{2} \text{ units square}$$

$$(5) f(x) = 0 \quad \text{يتقاطع منحنى الدالة مع محور السينات إذا كان:}$$

$$\Rightarrow x(x^2 - 6) = 0 \Rightarrow x = -\sqrt{6}, x = 0, x = \sqrt{6} \quad \text{فيكون:}$$

$$A = \int_0^{\sqrt{6}} (-x^3 + 6x) dx + \int_{\sqrt{6}}^3 (x^3 - 6x) dx = \left[-\frac{x^4}{4} + 3x^2 \right]_0^{\sqrt{6}} + \left[\frac{x^4}{4} - 3x^2 \right]_{\sqrt{6}}^3 = \frac{45}{4} \text{ units square}$$

$$(6) A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x dx = \left[\frac{1}{2} \sin 2x \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - \left[\frac{1}{2} \sin 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{3}{2} \text{ units square}$$

$$(7) A = \int_0^2 (x^2 + x^2 - 4x + 5) dx = \int_0^2 (2x^2 - 4x + 5) dx = \left[2\frac{x^3}{3} - 2x^2 + 5x \right]_0^2 = \frac{22}{3} \text{ units square}$$

$$(8) A = \int_1^8 (x - \sqrt[3]{x}) dx = \left[\frac{x^2}{2} - \frac{3}{4}x^{\frac{4}{3}} \right]_1^8 = \frac{81}{4} \text{ units square}$$

(9) حل $3 - x = 2x^2$ ، إذا تقاطع المنحنيات عند $x = 1$

$$A = \int_0^1 (3 - x - 2x^2) dx + \int_1^3 (2x^2 - 3 + x) dx$$

$$= \left[3x - \frac{x^2}{2} - \frac{2}{3}x^3 \right]_0^1 + \left[\frac{2}{3}x^3 - 3x + \frac{x^2}{2} \right]_1^3 = \frac{103}{6} \text{ units square}$$

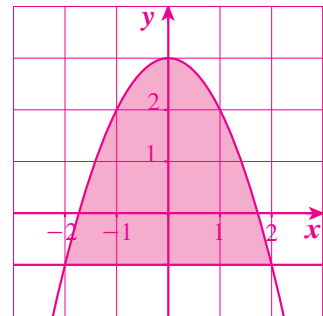
(10) تقاطع المنحنيات عند $x = \pm 2$

استخدم التناظر:

$$A = 2 \int_0^2 (3 - x^2 + 1) dx = 2 \int_0^2 (4 - x^2) dx$$

$$= 2 \left[4x - \frac{1}{3}x^3 \right]_0^2 = 2 \left[\left(8 - \frac{8}{3} \right) - 0 \right]$$

$$= \frac{32}{3} \text{ units square}$$



حل آخر

$$f(x) > g(x)$$

$$A = \int_{-2}^2 (f(x) - g(x)) dx = \int_{-2}^2 (3 - x^2 + 1) dx$$

$$= \left[3x - \frac{x^3}{3} + x \right]_{-2}^2 = \left(6 - \frac{8}{3} + 2 \right) - \left(-6 + \frac{8}{3} - 2 \right) = \frac{32}{3} \text{ units square}$$

(11) حل $x^2 - 2 = 2$ ؛ $x^2 = 4$ ، إذا تقاطع المنحنيات عند $x = \pm 2$

$$A = \int_{-2}^2 [2 - (x^2 - 2)] dx = \int_{-2}^2 (4 - x^2) dx = \left[4x - \frac{1}{3}x^3 \right]_{-2}^2 = \left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right) = \frac{32}{3} \text{ units square}$$

(12) حل $2x - x^2 = -2x$ ؛ $4x - x^2 = 0$

إذاً، تقاطع المنحنيات عند $x = 0$ ، $x = 4$

$$A = \int_0^4 (4x - x^2) dx = \left[2x^2 - \frac{1}{3}x^3 \right]_0^4 = \frac{32}{3} \text{ units square}$$

(13) حل $x^2 = 1$ ؛ $7 - 2x^2 = x^2 + 4$ ، إذاً تقاطع المنحنيات عند $x = \pm 1$

$$A = \int_{-1}^1 [(7 - 2x^2) - (x^2 + 4)] dx = \int_{-1}^1 (-3x^2 + 3) dx = 3 \int_{-1}^1 (1 - x^2) dx$$

$$= 3 \left[x - \frac{1}{3}x^3 \right]_{-1}^1 = 3 \left[\frac{2}{3} - \left(-\frac{2}{3} \right) \right] = 4 \text{ units square}$$

المجموعة B تمارين موضوعية

- (1) (b) (2) (a) (3) (a) (4) (b) (5) (b) (6) (d)
 (7) (b) (8) (c) (9) (a) (10) (a)

تمرّن 2-6

حجوم الأجسام الدورانية

المجموعة A تمارين مقالية

$$V = \int_0^2 \pi x^4 dx = \left[\pi \frac{1}{5} x^5 \right]_0^2 = \frac{32\pi}{5} \text{ units cube} \quad \text{(1) الحجم:}$$

$$V = \pi \int_1^4 \frac{1}{x^2} dx = \pi \left[-\frac{1}{x} \right]_1^4 = \frac{3\pi}{4} \text{ units cube} \quad \text{(2) الحجم:}$$

$$(3) V = \pi \int_{-1}^1 (1 - x^2) dx = 2\pi \left[x - \frac{x^3}{3} \right]_0^1 = \frac{4}{3} \pi \text{ units cube}$$

(4) نقاط التقاطع عند $x = -1$ ، $x = 2$

$$V = \pi \int_{-1}^2 [(x+3)^2 - (x^2+1)^2] dx = \pi \int_{-1}^2 (-x^4 - x^2 + 6x + 8) dx$$

$$= \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} + 3x^2 + 8x \right]_{-1}^2 = \frac{117\pi}{5} \text{ units cube}$$

(5) تمتد المنطقة المظللة من $x = -\frac{\pi}{4}$ إلى $x = \frac{\pi}{4}$

$$V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (2 - \sec^2 x) dx = \pi \left[2x - \tan x \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \pi^2 - 2\pi \text{ units cube}$$

(6) تمتد المنطقة المظللة من $x = 1$ إلى $x = 4$

$$V = \pi \int_1^4 ((x+1)^2 - (x-1)^2) dx = \pi \int_1^4 4x dx = \left[2\pi x^2 \right]_1^4 = 30\pi \text{ units cube}$$

(7) تمتد المنطقة المظللة من $x = 0$ إلى $x = 1$

$$V = \pi \int_0^1 (1 - x^2) dx = \pi \left[x - \frac{x^3}{3} \right]_0^1 = 2 \frac{\pi}{3} \text{ units cube}$$

(8) تمتد المنطقة المظللة من $x = 0$ إلى $x = 4$

$$V = \pi \int_0^4 x dx \implies V = \pi \left[\frac{x^2}{2} \right]_0^4 = 8\pi \text{ units cube}$$

$$(9) V = \pi \int_0^h \left(\frac{r}{h} x \right)^2 dx = \pi \frac{r^2}{h^2} \int_0^h x^2 dx = \pi \frac{r^2}{h^2} \left[\frac{x^3}{3} \right]_0^h$$

$$V = \pi \frac{r^2}{h^2} \times \frac{h^3}{3} = \frac{1}{3} \pi r^2 h \text{ units cube}$$

المجموعة B تمارين موضوعية

- (1) (b) (2) (a) (3) (b) (4) (a) (5) (c) (6) (d)
(7) (d) (8) (c) (9) (a) (10) (c) (11) (d) (12) (d)

تمرّن 3-6

طول قوس ومعادلة منحنى دالة

المجموعة A تمارين مقالية

$$(1) f'(x) = 3x^{\frac{1}{2}}$$

$$L = \int_0^{\frac{1}{3}} \sqrt{1 + (3x^{\frac{1}{2}})^2} dx = \int_0^{\frac{1}{3}} \sqrt{1 + 9x} dx = \frac{2}{27} \left[(1 + 9x)^{\frac{3}{2}} \right]_0^{\frac{1}{3}} = \frac{14}{27} \text{ units}$$

$$(2) f'(x) = 2(7 + 4x)^{\frac{1}{2}}$$

$$L = \int_1^{\frac{5}{4}} \sqrt{1 + 4(7 + 4x)} dx = \int_1^{\frac{5}{4}} \sqrt{29 + 16x} dx = \left[\frac{(29 + 16x)^{\frac{3}{2}}}{24} \right]_1^{\frac{5}{4}}$$

$$L = \frac{343 - 135\sqrt{5}}{24} \approx 1.714 \text{ units}$$

$$(3) f'(x) = \frac{1}{2}x^2 - \frac{1}{2x^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2} \right)^2} dx = \int_1^2 \sqrt{\left(\frac{1}{2}x^2 + \frac{1}{2x^2} \right)^2} dx = \int_1^2 \left(\frac{1}{2}x^2 + \frac{1}{2x^2} \right) dx$$

$$L = \left[\frac{1}{6}x^3 - \frac{1}{2x} \right]_1^2 = \frac{17}{12} \text{ units}$$

$$(4) f(x) = -\frac{x^3}{3} + x^2 - 4x + 19$$

$$(5) f(x) = -x^4 + x^2 + 5x - 2$$

$$(6) f(x) = \frac{1}{2} \sin 2x + 3$$

$$(7) f(x) = -\frac{1}{3} \cos 3x + 1$$

$$(8) f(x) = -\frac{1}{2} \ln|2x+5| + 3$$

$$(9) f'(x) = 4x^3 - 12x^2 - x + C_1$$

$$f'\left(-\frac{1}{2}\right) = 0 \Rightarrow C_1 = 3$$

$$f(x) = x^4 - 4x^3 - \frac{x^2}{2} + 3x + C_2$$

$$f\left(-\frac{1}{2}\right) = \frac{15}{16} \Rightarrow C_2 = 2$$

$$\Rightarrow f(x) = x^4 - 4x^3 - \frac{x^2}{2} + 3x + 2$$

المجموعة B تمارين موضوعية

$$(1) (b)$$

$$(2) (b)$$

$$(3) (b)$$

$$(4) (a)$$

$$(5) (c)$$

$$(6) (b)$$

$$(7) (b)$$

$$(8) (c)$$

$$(9) (d)$$

تمرن 4-6

المعادلات التفاضلية

المجموعة A تمارين مقالية

$$(1) y' = y'' = 3e^x \Rightarrow 3e^x - 3e^x + 2x = 2x \Rightarrow 2x = 2x$$

إذا الدالة $y = 3e^x$ هي حل للمعادلة التفاضلية $y'' - y' + 2x = 2x$

$$(2) y' = y'' = e^x \Rightarrow e^x + e^x = 2e^x$$

إذا الدالة $y = e^x$ هي حل للمعادلة التفاضلية $y + y'' = 2e^x$

$$(3) y = \int (x^2 + x + 2) dx = \frac{x^3}{3} + \frac{x^2}{2} + 2x + C$$

$$y(1) = 4$$

$$\therefore C = \frac{7}{6}$$

$$\therefore y = \frac{x^3}{3} + \frac{x^2}{2} + 2x + \frac{7}{6}$$

$$(4) y' = \frac{1}{x} - x$$

$$\therefore y = \ln|x| - \frac{x^2}{2} + C$$

$$(5) y = 4 \ln|x| + C$$

$$\therefore C = 1$$

$$\therefore y = 4 \ln|x| + 1$$

$$(6) y(x) = ke^{3x}$$

$$(7) y = ke^{5x}$$

$$(8) y = ke^{\frac{5}{2}x}$$

$$\therefore k = 4e^{-5}$$

$$\therefore y = 4e^{\frac{5}{2}x-5}$$

$$(9) y = ke^{-\frac{1}{\sqrt{2}}x}$$

$$y(0) = \sqrt{2}$$

$$\therefore k = \sqrt{2}$$

$$\therefore y = \sqrt{2}e^{-\frac{1}{\sqrt{2}}x}$$

$$(10) y = ke^x - 1$$

$$(11) y = ke^{-8x} + \frac{1}{4}$$

$$ke^{-8x} + \frac{1}{4}$$

$$k = \frac{e^2}{2}$$

$$\therefore y = \frac{1}{2}e^{2-8x} + \frac{1}{4}$$

$$(12) y = ke^{-\frac{1}{2}x} + 4$$

$$y(0) = 2$$

$$\therefore k = -2$$

$$\therefore y = -2e^{-\frac{1}{2}x} + 4$$

$$(13) y' = \cos 4x + C_1$$

$$y = \frac{1}{4}\sin 4x + C_1x + C_2$$

$$(14) y' = 3x^2 - 8x + C_1$$

$$y = x^3 - 4x^2 + C_1x + C_2$$

$$(15) y = C_1e^{\frac{5}{2}x} + C_2e^{-3x}$$

$$(16) y = (C_1 + C_2x)e^{3x}$$

$$(17) y = C_1 \cos 3x + C_2 \sin 3x$$

$$(18) y = (C_1x + C_2)e^x$$

$$(19) y = e^{-x} \left(C_1 \cos \frac{\sqrt{2}}{2}x + C_2 \sin \frac{\sqrt{2}}{2}x \right)$$

(20) (a) $y = ke^{-2x}$

(b) $k = \frac{1}{2} \therefore y = \frac{1}{2}e^{-2x}$

المجموعة B تمارين موضوعية

- (1) (a) (2) (b) (3) (b) (4) (b) (5) (a) (6) (a)
(7) (a) (8) (c) (9) (b) (10) (c) (11) (c) (12) (d)
(13) (a) (14) (d)

اختبار الوحدة السادسة

(1) تمتد المنطقة المظللة من $x = 0$ إلى $x = 1$.

$$A = \int_0^1 (x^2 - 4x + 3)dx = \left[\frac{x^3}{3} - 2x^2 + 3x \right]_0^1 = \frac{4}{3} \text{ units square}$$

(2) تمتد المنطقة المظللة من $x = 1$ إلى $x = 5$.

$$A = \int_1^5 (-x^2 + 6x - 5)dx = \left[-\frac{x^3}{3} + 3x^2 - 5x \right]_1^5 = \frac{32}{3} \text{ units square}$$

(3) $A = \int_{-2}^0 (x^3 - 4x)dx + \left| \int_0^2 (x^3 - 4x)dx \right| = \left[\frac{x^4}{4} - 2x^2 \right]_{-2}^0 - \left[\frac{x^4}{4} - 2x^2 \right]_0^2 = 4 + 4 = 8 \text{ units square}$

(4) $A = \int_1^2 (x^2 + 1 - \sqrt{x})dx = \left[\frac{x^3}{3} + x - \frac{2}{3}x\sqrt{x} \right]_1^2 = 4 - \frac{4}{3}\sqrt{2} \text{ units square}$

(5) يتقاطع المنحنيات عند النقطة: $x = -1$ ، $x = 0$ و $x = 1$.

$$A = \int_{-1}^0 (x^3 + 1 - x - 1)dx + \int_0^1 (x + 1 - x^3 - 1)dx = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} \text{ units square}$$

(6) يتقاطع المنحنيات عند $x = 2$ و $x = -2$.

$$V = \pi \int_{-2}^2 \left(4 - \frac{1}{4}x^4 \right) dx = \pi \left[4x - \frac{1}{20}x^5 \right]_{-2}^2 = \frac{64}{5} \pi \text{ units cube}$$

(7) تمتد المنطقة المظللة من $x = -1$ إلى $x = 2$ ويتقاطعا عند النقطة $x = \frac{1}{2}$.

$$V = \pi \int_{-1}^{\frac{1}{2}} [(-x+3)^2 - (x+2)^2] dx + \pi \int_{\frac{1}{2}}^2 [(x+2)^2 - (-x+3)^2] dx = \int_{-1}^{\frac{1}{2}} (5-10x) dx + \int_{\frac{1}{2}}^2 (-5+10x) dx$$
$$= [5x - 5x^2]_{-1}^{\frac{1}{2}} + [-5x + 5x^2]_{\frac{1}{2}}^2 = \frac{135}{4} \pi \text{ units cube}$$

(8) $V = \pi \int_{-2}^1 [(-x^2 + 4)^2 - (x + 2)^2] dx = \pi \int_{-2}^1 (x^4 - 9x^2 - 4x + 12) dx$

$$V = \pi \left[\frac{x^5}{5} - 3x^3 - 2x^2 + 12x \right]_{-2}^1 = \frac{108}{5} \pi \text{ units cube}$$

(9) $f'(x) = \frac{1}{2}x^{\frac{1}{2}}$

$$L = \int_0^{12} \sqrt{1 + \frac{1}{4}x} dx = \frac{8}{3} \left[\left(1 + \frac{1}{4}x \right)^{\frac{3}{2}} \right]_0^{12} = \frac{56}{3} \text{ units}$$

$$(10) f'(x) = -\sqrt{3}$$

$$L = \int_{-3}^1 \sqrt{1+3} dx = \int_{-3}^1 2 dx = 8 \text{ units}$$

$$(11) f'(x) = (-1+2x)^{\frac{1}{2}}$$

$$L = \int_2^8 \sqrt{1+(-1+2x)} dx = \int_2^8 \sqrt{2x} dx = \frac{56}{3} \text{ units}$$

$$(12) f'(x) = 3x^2 - 2x + 1 \quad \therefore f(x) = x^3 - x^2 + x + C$$

يمر بالنقطة $A(-1, -5)$ $C = -2$ \therefore

$$\therefore f(x) = x^3 - x^2 + x - 2$$

$$(13) f'(x) = \frac{-1}{3x-2} \quad \therefore f(x) = -\frac{1}{3} \ln|3x-2| + C$$

يمر بالنقطة $A(1, -1)$ $C = -1$ \therefore

$$\therefore f(x) = -\frac{1}{3} \ln|3x-2| - 1$$

(14) $A(-1, 3)$ نقطة صغرى محلية إذاً:

$$f'(-1) = 0$$

$$f'(x) = 4x^3 - 4x + C_1 \quad \therefore C_1 = 0$$

$$f(x) = x^4 - 2x^2 + C_2$$

منحنى f يمر بالنقطة $A(-1, 3)$ إذاً:

$$C_2 = 4$$

$$\therefore f(x) = x^4 - 2x^2 + 4$$

$$(15) y = ke^{-\frac{5}{3}x} + \frac{2}{5}$$

$$(16) y = k|x|^{\frac{5}{3}}$$

$$(17) y = C_1 e^{3x} + C_2 e^{4x}$$

$$(18) y = (C_1 + C_2 x)e^{3x}$$

$$(19) y = e^{-2x}(C_1 \cos 4x + C_2 \sin 4x)$$

$$(20) y = C_1 \cos 4x + C_2 \sin 4x$$

تمارين إثرائية

$$(1) A = \int_0^{\pi} (1 - \cos^2 x) dx = \int_0^{\pi} \sin^2 x dx = \left[\frac{1}{2}x - \frac{1}{4}\sin 2x \right]_0^{\pi} = \frac{\pi}{2} \text{ units square} \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

(2) استخدم التناظر:

$$2 \int_0^2 [2x^2 - (x^4 - 2x^2)] dx = 2 \int_0^2 (-x^4 + 4x^2) dx = 2 \left[-\frac{1}{5}x^5 + \frac{4}{3}x^3 \right]_0^2 = 2 \left[\left(-\frac{32}{5} + \frac{32}{3} \right) - 0 \right]$$

$$= \frac{128}{15} \text{ units square}$$

(3) استخدم التناظر:

$$2 \int_0^1 (x^2 + 2x^4) dx = 2 \left[\frac{1}{3}x^3 + \frac{2}{5}x^5 \right]_0^1 = 2 \left(\frac{1}{3} + \frac{2}{5} \right) = \frac{22}{15} \text{ units square}$$

$$(4) A = \int_{-2}^2 (8 + 2x^2 - x^4) dx = \frac{448}{15} \approx 32.5\bar{3} \text{ units square}$$

$$(5) A(x) = \int_{-3}^5 (15 + 2x - x^2) dx = \frac{256}{3} \text{ units square}$$

(6) يتقاطع منحنيا x ، $g(x) = x$ ، $f(x) = \frac{1}{x^2}$ عند $x = 1$ ومنه تكون:

$$A = \int_0^1 x dx + \int_1^2 \frac{1}{x^2} dx = \left[\frac{1}{2}x^2 \right]_0^1 + \left[-\frac{1}{x} \right]_1^2 = \frac{1}{2} + \left[-\frac{1}{2} - (-1) \right] = 1 \text{ units square}$$

$$(7) V = \pi \int_0^2 \left(\frac{2-x}{2} \right)^2 dx = \frac{\pi}{4} \left(-\frac{1}{3} \right) [(2-x)^3]_0^2 = \frac{2\pi}{3} \text{ units cube}$$

$$(8) V = \pi \int_0^{\frac{\pi}{2}} (\sin^2 \cos^2 x) dx$$

$$= \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \sin^2(2x) dx$$

$$= \frac{\pi}{8} \int_0^{\frac{\pi}{2}} (1 - \cos 4x) dx$$

$$= \frac{\pi}{8} \left[x - \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi^2}{16} \text{ units cube}$$

$$(9) f(x) = -\frac{1}{3} \cos 3x + C$$

$$\frac{4}{3} = -\frac{1}{3} \cos 3 \times \frac{\pi}{3} + C$$

$$\therefore C = 1$$

$$f(x) = -\frac{1}{3} \cos 3x + 1$$

$$(10) f'(x) = \frac{3}{4} x^{\frac{1}{2}}$$

$$L = \int_0^{27} \sqrt{1 + \frac{9}{16}x} dx = \frac{32}{27} \left[\frac{259}{64} \sqrt{259} - 1 \right] \approx 76 \text{ units}$$

$$(11) y(x) = Ae^{-\frac{3}{2}x} + \frac{4}{3}$$

$$(12) y(x) = A \sin(x) + B \cos(x)$$

(13) $y(x) = Ae^x + Be^{-x}$

(14) (a) $y = ke^{ax} + 2$

(b) $k = 168 \therefore y = 168e^{ax} + 2$

(c) $7 = 168e^{6a} \implies a = -\frac{\ln 24}{6}$

(15) $f'(x) = 3x^2 - 6x + C_1$

$A(3, -2)$ نقطة حرجة لمنحنى الدالة f إذاً:

$f'(3) = 0 \therefore C_1 = -9$

$f(x) = x^3 - 3x^2 - 9x + C_2$

$A(3, -2)$ هي نقطة على منحنى الدالة f إذاً:

$f(3) = -2 \therefore C_2 = 25$

$\therefore f(x) = x^3 - 3x^2 - 9x + 25$



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المجموعة A تمارين مقالية

(1) $y^2 = -12x$

(2) $x^2 = -8y$

(3) $x^2 = 8y$

(5) $y^2 = 2x$ البؤرة $(\frac{1}{2}, 0)$

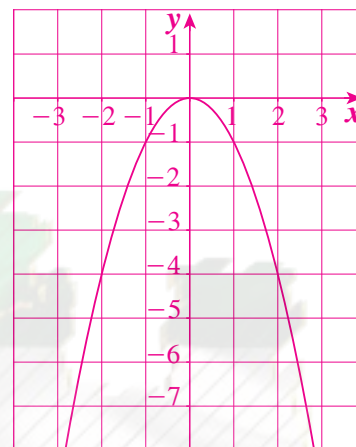
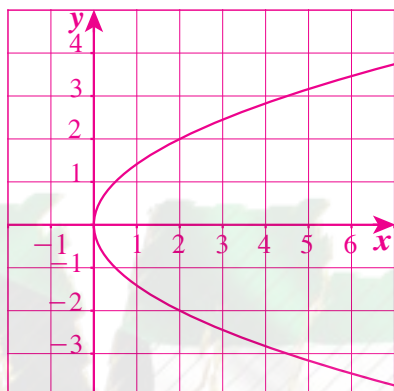
(4) $x^2 = -y$ البؤرة $(0, -\frac{1}{4})$

الدليل: $x = -\frac{1}{2}$

الدليل: $y = \frac{1}{4}$

خطّ التماثل محور السينات

خطّ التماثل محور الصادات



(7) $y^2 = -\frac{x}{8}$ البؤرة $(-\frac{1}{32}, 0)$

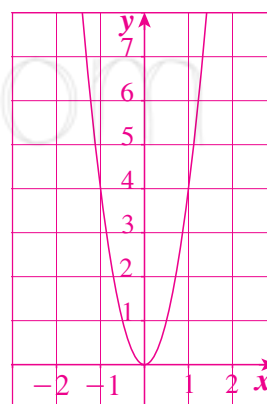
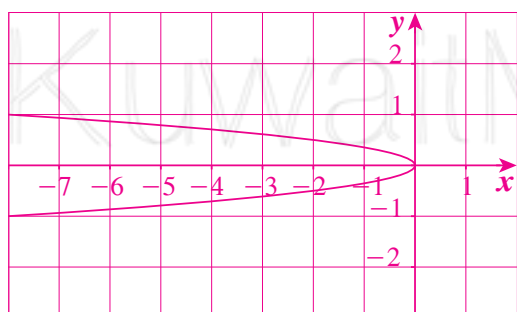
(6) $x^2 = \frac{1}{4}y$ البؤرة $(0, \frac{1}{16})$

الدليل: $x = \frac{1}{32}$

الدليل: $y = -\frac{1}{16}$

خطّ التماثل محور السينات

خطّ التماثل محور الصادات



(8) معادلة القطع المكافئ هي: $y^2 = 4px$

وبالتعويض عن (x, y) بإحداثيات A نحصل على:

$(2^2) = 4p(-1)$

$4 = -4p$

$p = -1$

$y^2 = -4x$

المعادلة:

(9) النقطتان $A(-3, 4)$, $B(3, 4)$ متمائلتان في محور الصادات

$$x^2 = 4py \text{ هي: معادلة القطع المكافئ}$$

وبالتعويض عن (x, y) بإحداثيات A (أو بإحداثيات B) نحصل على:

$$(-3)^2 = 4p(4)$$

$$9 = 16p \implies p = \frac{9}{16}$$

المعادلة:

$$x^2 = 4 \times \frac{9}{16}y$$

$$x^2 = \frac{9}{4}y$$

(10) البؤرة $(-4 ; 0)$ إذا المعادلة هي: $x^2 = -16y$

(11) البؤرة $(5 ; 0)$ إذا المعادلة هي: $y^2 = 20x$

$$(12) \quad x^2 = 10y \quad p = \frac{10}{4} = \frac{5}{2} \quad \text{إذا البؤرة } \left(0, \frac{5}{2}\right)$$

(13) معادلة القطع المكافئ هي على الصورة: $x^2 = 4py$

لنأخذ النقطة $A(50, 15)$ وبالتعويض عن (x, y) بإحداثيات A نحصل على:

$$(50)^2 = 4p(15)$$

$$2500 = 60p$$

$$p = \frac{125}{3}$$

المعادلة:

$$x^2 = \frac{500}{3}y$$

الإحداثي السيني للدعامة: $50 - 8 = 42$

بالتعويض في المعادلة نوجد y : $(42)^2 = \frac{500}{3}y$ ومنه $y \approx 10.6$

طول الدعامة يكون: $10.6 + 5 = 15.6 \text{ m}$

المجموعة B تمارين موضوعية

- | | | | | |
|----------|----------|----------|----------|----------|
| (1) (a) | (2) (b) | (3) (a) | (4) (b) | (5) (a) |
| (6) (b) | (7) (a) | (8) (d) | (9) (c) | (10) (d) |
| (11) (a) | (12) (b) | (13) (c) | (14) (a) | (15) (b) |
| (16) (c) | (17) (b) | (18) (d) | | |

المجموعة A تمارين مقالية

$$(1) \frac{x^2}{8^2} + \frac{y^2}{6^2} = 1$$

$$a^2 = 8^2 \Rightarrow a = 8$$

رأسا القطع: $A_1(-8, 0)$, $A_2(8, 0)$

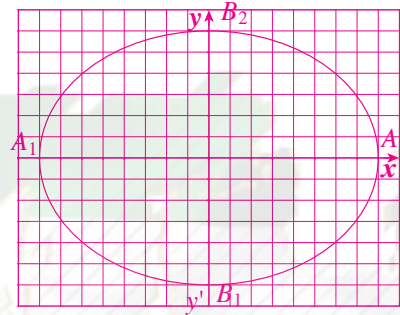
$$b^2 = 6^2 \Rightarrow b = 6$$

النقطتان الطرفيتان للمحور الأصغر: $B_1(0, -6)$, $B_2(0, 6)$

$$a^2 = b^2 + c^2 \Rightarrow c^2 = a^2 - b^2$$

$$c^2 = 8^2 - 6^2 = 28 \Rightarrow c = 2\sqrt{7}$$

البؤرتان: $F_1(-2\sqrt{7}, 0)$, $F_2(2\sqrt{7}, 0)$



$$x = \frac{a^2}{c} = \frac{64}{2\sqrt{7}} = \frac{32\sqrt{7}}{7}, \quad x = -\frac{a^2}{c} = \frac{-64}{2\sqrt{7}} = \frac{-32\sqrt{7}}{7}$$

معادلتنا دليلي القطع الناقص: طول المحور الأكبر: $2a = 2 \times 8 = 16$

طول المحور الأصغر: $2b = 2 \times 6 = 12$

$$(2) \frac{x^2}{4^2} + \frac{y^2}{6^2} = 1$$

$$a^2 = 6^2 \Rightarrow a = 6$$

رأسا القطع: $A_1(0, -6)$, $A_2(0, 6)$

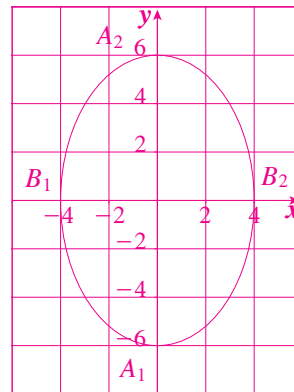
$$b^2 = 4^2 \Rightarrow b = 4$$

النقطتان الطرفيتان للمحور الأصغر: $B_1(-4, 0)$, $B_2(4, 0)$

$$a^2 = b^2 + c^2 \Rightarrow c^2 = a^2 - b^2$$

$$c^2 = 6^2 - 4^2 = 20 \Rightarrow c = 2\sqrt{5}$$

البؤرتان: $F_1(0, -2\sqrt{5})$, $F_2(0, 2\sqrt{5})$



$$y = \frac{a^2}{c} = \frac{36}{2\sqrt{5}} = \frac{18\sqrt{5}}{5} \quad y = -\frac{a^2}{c} = \frac{-36}{2\sqrt{5}} = \frac{-18\sqrt{5}}{5} \quad \text{معادلنا دليلي القطع الناقص:}$$

$$2a = 12 \quad \text{طول المحور الأكبر:}$$

$$2b = 8 \quad \text{طول المحور الأصغر:}$$

$$(3) \quad 3x^2 + 5y^2 - 225 = 0$$

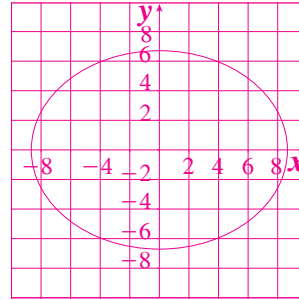
$$\frac{3x^2}{225} + \frac{5y^2}{225} = \frac{225}{225}$$

$$\frac{x^2}{75} + \frac{y^2}{45} = 1 \quad \text{معادلة القطع الناقص:}$$

$$a^2 = 75 \implies a = 5\sqrt{3}$$

$$A_1(5\sqrt{3}, 0), A_2(-5\sqrt{3}, 0) \quad \text{رأسا القطع:}$$

$$b^2 = 45 \implies b = 3\sqrt{5}$$



$$B_1(0, -3\sqrt{5}), B_2(0, 3\sqrt{5}) \quad \text{النقطتان الطرفيتان للمحور الأصغر:}$$

$$a^2 = b^2 + c^2 \implies c^2 = a^2 - b^2$$

$$c^2 = 75 - 45 = 30 \implies c = \sqrt{30}$$

$$F_1(-\sqrt{30}, 0), F_2(\sqrt{30}, 0) \quad \text{البؤرتان:}$$

$$x = \frac{a^2}{c} = \frac{75}{\sqrt{30}} = \frac{5\sqrt{30}}{2} \quad x = -\frac{a^2}{c} = \frac{-75}{\sqrt{30}} = \frac{-5\sqrt{30}}{2} \quad \text{معادلنا دليلي القطع:}$$

$$2a = 10\sqrt{3} \quad \text{طول المحور الأكبر:}$$

$$2b = 6\sqrt{5} \quad \text{طول المحور الأصغر:}$$

$$(4) \quad 4x^2 + y^2 - 28 = 0$$

$$\frac{4x^2}{28} + \frac{y^2}{28} = \frac{28}{28}$$

$$\frac{x^2}{7} + \frac{y^2}{28} = 1 \quad \text{معادلة القطع الناقص:}$$

$$a^2 = 28 \implies a = 2\sqrt{7}$$

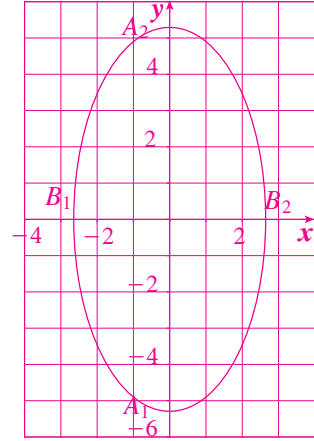
$$A_1(0, -2\sqrt{7}), A_2(0, 2\sqrt{7}) \quad \text{رأسا القطع:}$$

النقطتان الطرفيتان للمحور الأصغر: $B_1(-\sqrt{7}, 0)$, $B_2(\sqrt{7}, 0)$

$$a^2 = b^2 + c^2 \implies c^2 = a^2 - b^2$$

$$c^2 = 28 - 7 = 21 \implies c = \sqrt{21}$$

البؤرتان: $F_1(0, -\sqrt{21})$, $F_2(0, \sqrt{21})$



معادلتنا دليلي القطع الناقص:

$$y = \frac{a^2}{c} = \frac{28}{\sqrt{21}} = \frac{28\sqrt{21}}{21} = \frac{4}{3}\sqrt{21}$$

$$y = -\frac{a^2}{c} = \frac{-28}{\sqrt{21}} = \frac{-28\sqrt{21}}{21} = -\frac{4}{3}\sqrt{21}$$

طول المحور الأكبر: $2a = 4\sqrt{7}$

طول المحور الأصغر: $2b = 2\sqrt{7}$

(5) $c = 2$, $b = 3$

$$a^2 = b^2 + c^2 = 3^2 + 2^2 = 13$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \therefore \quad \frac{x^2}{13} + \frac{y^2}{9} = 1$$

(6) $2a = 10 \implies a = 5$; $c = 3$

$$b^2 = a^2 - c^2 = 25 - 9 = 16 \implies b = 4$$

$$\therefore \quad \frac{x^2}{25} + \frac{y^2}{16} = 1$$

(7) $a = 5$, $2b = 4 \implies b = 2$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1 \quad \text{فتكون معادلة القطع الناقص:}$$

(8) $b = 4$

$$2a = 10 \implies a = 5$$

$$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1 \implies \frac{x^2}{25} + \frac{y^2}{16} = 1$$

(9) $c = 5$

$$a^2 = b^2 + 5^2 \implies a^2 = b^2 + 25$$

$$\frac{2^2}{a^2} + \frac{3^2}{b^2} = 1$$

$$\Rightarrow a^2 b^2 = 4b^2 + 9a^2 \Rightarrow (b^2 + 25)b^2 = 4b^2 + 9(b^2 + 25) \Rightarrow b^4 + 25b^2 = 4b^2 + 9b^2 + 225$$

$$\Rightarrow b^4 + 12b^2 - 225 = 0 \Rightarrow b^2 = -6 + 3\sqrt{29}$$

$$\Rightarrow a^2 = 19 + 3\sqrt{29}$$

$$\frac{x^2}{(19 + 3\sqrt{29})} + \frac{y^2}{(-6 + 3\sqrt{29})} = 1$$

$$(10) \quad a = 6 ; b = 4$$

$$\frac{x^2}{36} + \frac{y^2}{16} = 1 \text{ : معادلة القطع الناقص}$$

$$(11) \quad c = 5 ; 2b = 6 \Rightarrow b = \frac{6}{2} = 3$$

$$a^2 = c^2 + b^2 \Rightarrow a^2 = 5^2 + 3^2 = 25 + 9 = 34$$

$$\frac{x^2}{34} + \frac{y^2}{9} = 1$$

$$(12) \quad 2a = 10 \Rightarrow a = 5 ; 2c = 6 \Rightarrow c = \frac{6}{2} = 3$$

$$b^2 = a^2 - c^2 = 25 - 9 = 16$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

المجموعة B تمارين موضوعية

- | | | | | | |
|----------|----------|----------|----------|----------|----------|
| (1) (a) | (2) (a) | (3) (a) | (4) (b) | (5) (a) | (6) (c) |
| (7) (a) | (8) (b) | (9) (d) | (10) (d) | (11) (b) | (12) (c) |
| (13) (b) | (14) (c) | (15) (d) | | | |

تمرن 3-7

القطع الزائد

المجموعة A تمارين مقالية

$$(1) \quad \frac{y^2}{25} - \frac{x^2}{16} = 1$$

$$A_1(0, 5) , A_2(0, -5)$$

$$a = 5 \therefore a^2 = 25 \therefore \text{رأسا القطع الزائد}$$

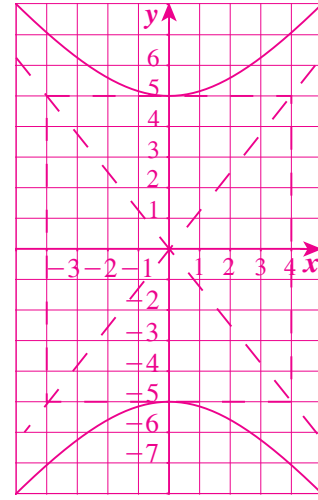
$$b^2 = 16 \Rightarrow b = 4$$

$$c^2 = a^2 + b^2 = 25 + 16 = 41 \Rightarrow c = \sqrt{41}$$

البؤرتان: $F_1(0, \sqrt{41})$, $F_2(0, -\sqrt{41})$

معادلتا الخطين المقاربتين: $y = \pm \frac{a}{b}x = \pm \frac{5}{4}x$

معادلتا الدليلين: $y = \pm \frac{a^2}{c} = \pm \frac{25\sqrt{41}}{41}$



طول المحور الأكبر: $2a = 4 \times 5 = 10$

طول المحور المرافق: $2b = 2 \times 4 = 8$

(2) $24x^2 - 12y^2 - 192 = 0$

$$\frac{24x^2}{192} - \frac{12y^2}{192} = \frac{192}{192}$$

$$\frac{x^2}{8} - \frac{y^2}{16} = 1$$

$$a^2 = 8 \implies a = \sqrt{8} = 2\sqrt{2}$$

رأسا القطع: $A_1(2\sqrt{2}, 0)$, $A_2(-2\sqrt{2}, 0)$

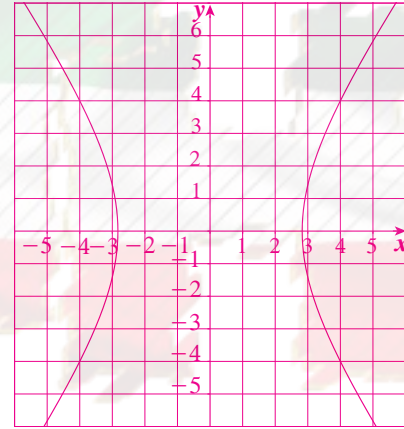
$$b^2 = 16 \implies b = 4$$

$$c^2 = a^2 + b^2 = 8 + 16 = 24 \implies c = 2\sqrt{6}$$

البؤرتان: $F_1(2\sqrt{6}, 0)$, $F_2(-2\sqrt{6}, 0)$

معادلتا الخطين المقاربتين: $y = \pm \frac{b}{a}x = \pm \frac{4x}{2\sqrt{2}} = \pm \sqrt{2}x$

معادلتا الدليلين: $x = \pm \frac{a^2}{c} = \pm \frac{8}{2\sqrt{6}} = \pm \frac{2\sqrt{6}}{3}$



طول المحور الأكبر: $2a = 4\sqrt{2}$

طول المحور المرافق: $2b = 2 \times 4 = 8$

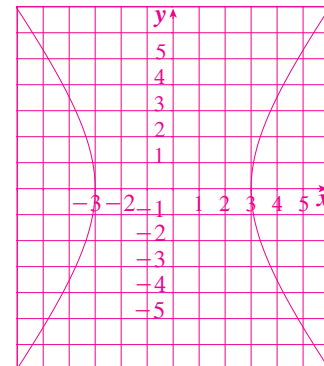
(3) $c = 5$, $a = 3$

$$c^2 = a^2 + b^2 = b^2 = c^2 - a^2$$

$$b^2 = 25 - 9 = 16 \implies b = 4$$

معادلة القطع الزائد: $\frac{x^2}{9} - \frac{y^2}{16} = 1$

معادلتا الخطين المقاربتين: $y = \pm \frac{b}{a}x = \pm \frac{4x}{3}$



$$(4) \frac{a}{b} = 2 \Rightarrow a = 2b$$

$$c = \sqrt{5}$$

$$c^2 = a^2 + b^2 \Rightarrow (2b^2) + b^2 = 5 \Rightarrow 5b^2 = 5$$

$$\Rightarrow b^2 = 1 \Rightarrow b = 1$$

$$\therefore a = 2$$

$$\Rightarrow \frac{y^2}{4} - \frac{x^2}{1} = 1, \frac{y^2}{4} - x^2 = 1 \quad \text{معادلة القطع الزائد.}$$

$$(5) a = \frac{2}{3}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\frac{4}{9}} - \frac{y^2}{b^2} = 1$$

لنضع إحداثيات النقطة (1, 1) في المعادلة:

$$\frac{1}{\frac{4}{9}} - \frac{1}{b^2} = 1 \Rightarrow b^2 = \frac{4}{5}$$

$$\Rightarrow \frac{x^2}{\frac{4}{9}} - \frac{y^2}{\frac{4}{5}} = 1 \quad \text{معادلة القطع الزائد.}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (6) \text{ بما أن محوره الأساسي هو جزء من محور السينات فالمعادلة هي:}$$

لنضع إحداثيات A في المعادلة:

$$\frac{4}{a^2} - \frac{1}{b^2} = 1$$

$$\frac{4}{a^2} = \frac{1}{b^2} + 1 \Rightarrow \frac{1}{a^2} = \frac{1}{4b^2} + \frac{1}{4}$$

لنضع إحداثيات B في المعادلة:

$$\frac{16}{a^2} - \frac{9}{b^2} = 1$$

بالتعويض نوجد المعادلة التالية:

$$16\left(\frac{1}{4b^2} + \frac{1}{4}\right) - \frac{9}{b^2} = 1$$

$$4 + \frac{4}{b^2} - \frac{9}{b^2} = 1 \Rightarrow \frac{5}{b^2} = 3 \Rightarrow b^2 = \frac{5}{3}$$

$$\frac{4}{a^2} - \frac{1}{\frac{5}{3}} = 1 \Rightarrow \frac{4}{a^2} = \frac{8}{5} \Rightarrow a^2 = \frac{5}{2}$$

$$\frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{5}{3}} = 1 \quad \text{المعادلة هي:}$$

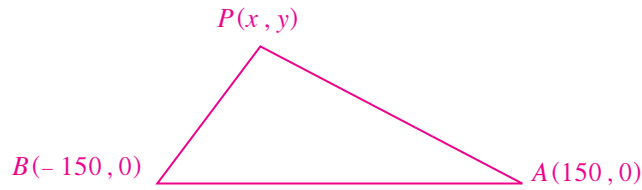
(7) نستخدم قاعدة المسافة بدلالة الزمن والسرعة:

$$d = vt \Leftrightarrow t = \frac{d}{v}$$

$$t_1 = \frac{PA}{50}$$

$$t_2 = \frac{PB}{50}$$

$$t_1 - t_2 = \frac{PA}{50} - \frac{PB}{50}$$



ولكن: $t_1 - t_2 = 2$

$$2 = \frac{PA}{50} - \frac{PB}{50} \implies PA - PB = 100$$

بما أن A, B نقطتان ثابتتان فيكون منحنى النقاط المتغيرة P هي قطع زائد بؤرتاه هما A, B حيث: $2a = 100$

$$c = 150, a = 50$$

$$b^2 = (150)^2 - (50)^2 = 20000$$

$$\frac{x^2}{2500} - \frac{y^2}{20000} = 1 \quad \text{معادلة القطع الزائد:}$$

المجموعة B تمارين موضوعية

- | | | | | |
|----------|----------|----------|----------|----------|
| (1) (a) | (2) (a) | (3) (b) | (4) (b) | (5) (c) |
| (6) (a) | (7) (d) | (8) (b) | (9) (c) | (10) (b) |
| (11) (a) | (12) (c) | (13) (a) | (14) (d) | |

تمرّن 4-7

الاختلاف المركزي

المجموعة A تمارين مقالية

$$(1) e = \frac{3}{2}, \frac{3}{2} > 1$$

إذا القطع المخروطي هو قطع زائد

$$c = 3, e = \frac{c}{a} \implies \frac{3}{a} = \frac{3}{2} \implies a = 2$$

ولكن في القطع الزائد:

$$c^2 = a^2 + b^2 \implies b^2 = 9 - 4$$

$$b^2 = 5$$

$$\frac{y^2}{5} - \frac{x^2}{4} = 1 \quad \text{معادلة القطع الزائد هي:}$$

$$(2) e = \frac{\sqrt{7}}{4}, \frac{\sqrt{7}}{4} < 1$$

إذا القطع المخروطي هو قطع ناقص

$$c = \sqrt{7}, e = \frac{c}{a} \implies \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{a} \implies a = 4$$

$$a^2 = b^2 + c^2 \implies b^2 = a^2 - c^2$$

$$b^2 = 16 - 7 \implies b^2 = 9$$

$$\frac{y^2}{16} + \frac{x^2}{9} = 1 \quad \text{معادلة القطع الناقص هي:}$$

$$(3) \quad e = \frac{5}{3}, \frac{5}{3} > 1$$

إذاً القطع المخروطي هو قطع زائد

$$a = 4, e = \frac{c}{a}$$

$$\frac{c}{4} = \frac{5}{3} \implies c = \frac{5 \times 4}{3} = \frac{20}{3}$$

$$c^2 = a^2 + b^2 \implies b^2 = c^2 - a^2 \quad \text{في القطع الزائد:}$$

$$b^2 = \frac{400}{9} - 16 = \frac{256}{9}$$

$$\frac{x^2}{16} - \frac{y^2}{\frac{256}{9}} = 1 \quad \text{المعادلة هي:}$$

$$(4) \quad e = \frac{3}{4}, \frac{3}{4} < 1$$

إذاً القطع المخروطي هو قطع ناقص

$$8 = \frac{a^2}{c} \implies c = \frac{a^2}{8} \quad \text{معادلة الدليل:}$$

$$\frac{3}{4} = \frac{c}{a} = \frac{\frac{a^2}{8}}{a} \implies \frac{3}{4} = \frac{a}{8} \implies a = 6$$

$$c = e \cdot a = \frac{3}{4} \times 6 = \frac{9}{2}$$

$$a^2 = b^2 + c^2 \implies b^2 = a^2 - c^2 \quad \text{في القطع الناقص:}$$

$$b^2 = 36 - \frac{81}{4} = \frac{63}{4}$$

$$\frac{x^2}{36} + \frac{y^2}{\frac{63}{4}} = 1 \quad \text{المعادلة هي:}$$

$$(5) \quad (a^2 = 9, b^2 = 4) \implies (a = 3, b = 2)$$

$$a^2 = b^2 + c^2 \quad \text{في القطع الناقص:}$$

$$c^2 = 9 - 4 = 5 \implies c = \sqrt{5}$$

$$e = \frac{c}{a} = \frac{\sqrt{5}}{3} \quad \text{الاختلاف المركزي للقطع الناقص:}$$

$$(6) \quad 4y^2 - 9x^2 = 36$$

$$\frac{y^2}{9} - \frac{x^2}{4} = 1 \quad \text{على الصورة } \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$a^2 = 9 \implies a = 3 \quad \text{بالمقارنة}$$

$$b^2 = 4 \implies b = 2$$

$$c^2 = a^2 + b^2 = 9 + 4 = 13 \implies c = \sqrt{13}$$

$$e = \frac{c}{a} = \frac{\sqrt{13}}{3} > 1$$

$$(7) \quad a^2 = 7 \implies a = \sqrt{7}$$

$$b^2 = 16 \implies b = 4$$

الرأسان: $A_1(-\sqrt{7}, 0)$, $A_2(\sqrt{7}, 0)$

$$c^2 = a^2 + b^2 = 7 + 16 \implies c = \sqrt{23}$$

البؤرتان: $F_1(-\sqrt{23}, 0)$, $F_2(\sqrt{23}, 0)$

$$e = \frac{c}{a} = \frac{\sqrt{23}}{\sqrt{7}} = \frac{\sqrt{161}}{7} \quad \text{الاختلاف المركزي:}$$

$$x = \pm \frac{a^2}{c} = \pm \frac{7}{\sqrt{23}} = \pm \frac{7\sqrt{23}}{23} \quad \text{معادلتنا الدليلين:}$$

$$(8) \quad a^2 = 16 \implies a = 4$$

$$b^2 = 4 \implies b = 2$$

الرأسان: $A_1(0, -4)$, $A_2(0, 4)$

$$c^2 = a^2 + b^2 = 16 + 4 = 20 \implies c = 2\sqrt{5}$$

البؤرتان: $F_1(0, -2\sqrt{5})$, $F_2(0, 2\sqrt{5})$

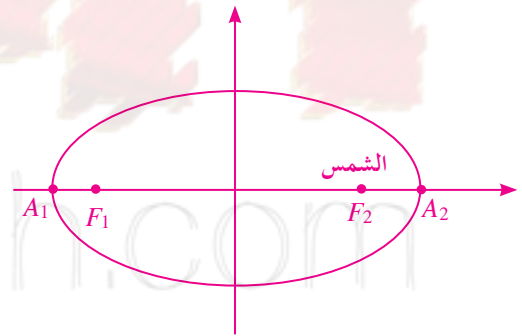
$$e = \frac{c}{a} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$$

$$y = \pm \frac{a^2}{c} = \pm \frac{16}{2\sqrt{5}} = \pm \frac{8\sqrt{5}}{5} \quad \text{معادلتنا الدليلين:}$$

$$(9) \quad 2a = 3000000 \implies a = 150000$$

$$e = \frac{c}{a} \implies c = e \cdot a = 0.017 \times 150000 = 2550$$

$$c = 2550$$



أصغر بعد للأرض عن الشمس هو: $F_2 A_2$ فيكون:

$$F_2 A_2 = 150000 - 2550 = 147450 \text{ km}$$

أكبر بعد للأرض عن الشمس هو: $F_2 A_1$ فيكون:

$$F_2 A_1 = 150000 + 2550 = 152550 \text{ km}$$

المجموعة B تمارين موضوعية

- | | | | | | |
|----------|----------|----------|----------|----------|----------|
| (1) (a) | (2) (b) | (3) (a) | (4) (b) | (5) (a) | (6) (a) |
| (7) (b) | (8) (b) | (9) (c) | (10) (d) | (11) (a) | (12) (c) |
| (13) (a) | (14) (b) | (15) (d) | (16) (a) | | |

اختبار الوحدة السابعة

(1) $4y^2 - 9x^2 = 36 \Rightarrow \frac{y^2}{9} - \frac{x^2}{4} = 1$

إذاً هي معادلة قطع زائد مركزه نقطة الأصل.

$$a^2 = 9, \quad b^2 = 4$$

$$\Rightarrow c^2 = 13 \Rightarrow c = \sqrt{13}$$

البؤرتان: $F_1(0, -\sqrt{13}), F_2(0, \sqrt{13})$

(2) $-2x^2 + 3y^2 + 10 = 0 \Rightarrow -2x^2 + 3y^2 = -10 \Rightarrow 2x^2 - 3y^2 = 10$

$$\frac{x^2}{5} - \frac{y^2}{\frac{10}{3}} = 1$$

إذاً هي معادلة قطع زائد مركزه نقطة الأصل.

$$a^2 = 5, \quad b^2 = \frac{10}{3}$$

$$\Rightarrow c^2 = \frac{25}{3} \Rightarrow c = \frac{5\sqrt{3}}{3}$$

البؤرتان: $F_1\left(-\frac{5\sqrt{3}}{3}, 0\right), F_2\left(\frac{5\sqrt{3}}{3}, 0\right)$

(3) $2x^2 + y^2 = 9$

$$\frac{x^2}{\frac{9}{2}} + \frac{y^2}{9} = 1$$

إذاً هي معادلة قطع ناقص مركزه نقطة الأصل.

$$a^2 = \frac{9}{2}, \quad b^2 = 9$$

$$c^2 = 9 - \frac{9}{2}$$

$$c^2 = \frac{9}{2}$$

البؤرتان: $F_1\left(0, -\frac{3\sqrt{2}}{2}\right), F_2\left(0, \frac{3\sqrt{2}}{2}\right)$

(4) $2x^2 - y^2 + 6 = 0 \Rightarrow 2x^2 - y^2 = -6 \Rightarrow y^2 - 2x^2 = 6$

$$\frac{y^2}{6} - \frac{x^2}{3} = 1$$

إذاً هي معادلة قطع زائد مركزه نقطة الأصل.

$$a^2 = 6, \quad b^2 = 3 \Rightarrow c^2 = 9$$

البؤرتان: $F_1(0, -3), F_2(0, 3)$

$$(5) \frac{x^2}{2^2} + \frac{y^2}{5^2} = 1.$$

هي معادلة قطع ناقص مركزه نقطة الأصل.

$$a^2 = 5^2 \Rightarrow a = 5$$

$$b^2 = 2^2 \Rightarrow b = 2$$

$$a^2 = b^2 + c^2 \Rightarrow c^2 = a^2 - b^2 \quad \text{في القطع الناقص:}$$

$$c^2 = 5^2 - 2^2 = 21 \Rightarrow c = \sqrt{21}$$

$$e = \frac{c}{a} = \frac{\sqrt{21}}{5} \quad \text{الاختلاف المركزي:}$$

$$F_1(0, -\sqrt{21}); F_2(0, \sqrt{21}) \quad \text{البؤرتان:}$$

$$y = \pm \frac{a^2}{c} = \pm \frac{25}{\sqrt{21}} = \pm \frac{25\sqrt{21}}{21} \quad \text{معادلتا الدليلين:}$$

$$(6) y^2 = 5x.$$

هي معادلة قطع مكافئ مركزه نقطة الأصل.

$$4p = 5 \Rightarrow p = \frac{5}{4}$$

$$e = 1 \quad \text{الاختلاف المركزي:}$$

$$F\left(\frac{5}{4}, 0\right) \quad \text{البؤرة:}$$

$$x = -\frac{5}{4} \quad \text{معادلة الدليل:}$$

$$(7) \frac{x^2}{4} - \frac{y^2}{9} = 1$$

هي معادلة قطع زائد مركزه نقطة الأصل.

$$a^2 = 4 \Rightarrow a = 2$$

$$b^2 = 9 \Rightarrow b = 3$$

$$c^2 = a^2 + b^2 = 13 \Rightarrow c = \sqrt{13} \quad \text{في القطع الزائد:}$$

$$e = \frac{c}{a} = \frac{\sqrt{13}}{2} \quad \text{الاختلاف المركزي:}$$

$$F_1(-\sqrt{13}, 0); F_2(\sqrt{13}, 0) \quad \text{البؤرتان:}$$

$$x = \pm \frac{a^2}{c} = \pm \frac{4}{\sqrt{13}} = \pm \frac{4\sqrt{13}}{13} \quad \text{معادلتا الدليلين:}$$

$$y = \pm \frac{b}{a}x = \pm \frac{3}{2}x \quad \text{معادلتا الخططين المقاربين:}$$

$$(8) \frac{x^2}{18^2} + \frac{y^2}{10^2} = 1$$

هي معادلة قطع ناقص مركزه نقطة الأصل.

$$a^2 = 18^2 \Rightarrow a = 18$$

$$b^2 = 10^2 \Rightarrow b = 10$$

في القطع الناقص: $a^2 = b^2 + c^2 = c^2 = a^2 - b^2$

$$c^2 = 18^2 - 10^2 = 224 \implies c = \sqrt{224} = 4\sqrt{14}$$

$$e = \frac{c}{a} = \frac{4\sqrt{14}}{18} = \frac{2\sqrt{14}}{9} \quad \text{الاختلاف المركزي:}$$

$$F_1(-4\sqrt{14}, 0); F_2(4\sqrt{14}, 0) \quad \text{البؤرتان:}$$

$$x = \pm \frac{a^2}{c} = \pm \frac{18^2}{4\sqrt{14}} = \pm \frac{81\sqrt{14}}{14} \quad \text{معادلتا الدليلين:}$$

$$(9) \quad y^2 = -3x$$

هي معادلة قطع مكافئ مركزه نقطة الأصل.

$$4p = -3 \implies p = -\frac{3}{4}$$

الاختلاف المركزي: $e = 1$

$$F\left(-\frac{3}{4}, 0\right) \quad \text{البؤرة:}$$

$$x = \frac{3}{4} \quad \text{معادلة الدليل:}$$

$$(10) \quad \frac{y^2}{16} - \frac{x^2}{9} = 1$$

هي معادلة قطع زائد مركزه نقطة الأصل.

$$a^2 = 16 \implies a = 4$$

$$b^2 = 9 \implies b = 3$$

في القطع الزائد: $c^2 = a^2 + b^2 = 16 + 9 = 25 \implies c = 5$

$$e = \frac{c}{a} = \frac{5}{4} \quad \text{الاختلاف المركزي:}$$

$$F_1(0, -5); F_2(0, 5) \quad \text{البؤرتان:}$$

$$x = \pm \frac{a^2}{c} = \pm \frac{16}{5} \quad \text{معادلتا الدليلين:}$$

$$y = \pm \frac{a}{b}x = \pm \frac{4}{3}x \quad \text{معادلتا الخططين المقاربين:}$$

$$(11) \quad x^2 + y^2 = r^2$$

$$\therefore \frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

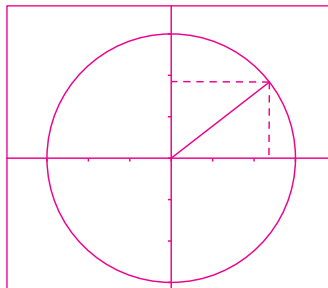
لتكن $M(x, y)$ نقطة على دائرة؛ لنذكر أن $OM = r$.

$$OM = \sqrt{x^2 + y^2}$$

$$\implies \sqrt{x^2 + y^2} = r$$

$$\implies (\sqrt{x^2 + y^2})^2 = r^2$$

$$\implies x^2 + y^2 = r^2$$



$$(12) \quad e = \frac{c}{a} = \frac{213125.9}{107124} \approx 1.99$$

$e = 1.99 > 1$ إذاً هي معادلة قطع زائد مركزه نقطة الأصل.

$$b^2 = c^2 - a^2 \implies b^2 = 3.39 \times 10^{10}$$

بفرض أن مركز القطع الزائد هو نقطة الأصل وأن المحور أفقي.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{تكون المعادلة:}$$

$$\implies \frac{x^2}{1.15 \times 10^{10}} - \frac{y^2}{3.39 \times 10^{10}} = 1$$

(13) لتكن $M(x, y)$ نقطة على القطع الزائد و $F_1(-155, 0)$ ، $F_2(155, 0)$ البؤرتين.

$$|MF_1 - MF_2| = 80$$

$$2a = 80 \implies a = 40 \implies a^2 = 1600$$

$$\therefore c = 155$$

$$b^2 = c^2 - a^2 \quad \text{في القطع الزائد:}$$

$$b^2 = 22425$$

$$\implies \frac{x^2}{1600} - \frac{y^2}{22425} = 1$$

$$(14) \quad (a) \quad e = \frac{\sqrt{2}}{2}, \quad \frac{\sqrt{2}}{2} < 1$$

إذاً هي معادلة قطع ناقص.

$$(b) \quad e = \frac{\sqrt{2}}{2} = \frac{c}{a} \implies 2c = \sqrt{2}a \implies a = \sqrt{2}c$$

$$x = 4 = \frac{a^2}{c} \implies 4 = \frac{(\sqrt{2}c)^2}{c} = 2c \implies c = 2 \implies a = 2\sqrt{2}$$

$$a^2 = b^2 + c^2 \implies (2\sqrt{2})^2 = b^2 + 4 \implies b^2 = 4 \implies b = 2 \quad \text{في القطع الناقص:}$$

(c) الصورة العامة للقطع الناقص حيث أن المحور القاطع ينطبق على محور السينات هي:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \implies \frac{x^2}{8} + \frac{y^2}{4} = 1$$

$$(15) \quad e = \frac{5}{4}, \quad \frac{5}{4} > 1 \quad \text{إذاً هي معادلة قطع زائد.}$$

$$\frac{5}{4} = \frac{c}{a} \implies 4c = 5a \implies a = \frac{4}{5}c$$

$$c = 5 \implies a = \frac{4}{5} \times 5 = 4$$

$$b^2 = c^2 - a^2 = 25 - 16 = 9 \quad \text{في القطع الزائد:}$$

$$\frac{y^2}{16} - \frac{x^2}{9} = 1 \quad \text{إذاً الصورة العامة للقطع الزائد هي:}$$

$$(16) \quad x^2 = -4y$$

$$(17) \quad y^2 = 8x$$

$$(18) \quad \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$(19) \frac{x^2}{16} + \frac{y^2}{9} = 1$$

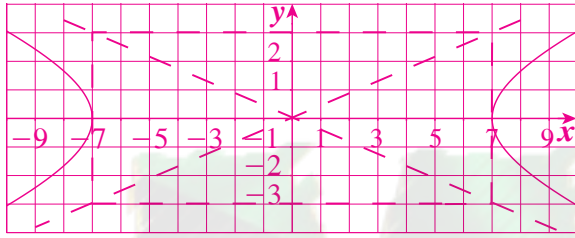
$$(20) 2a = 12 \implies a = 6$$

$$2c = 20 \implies c = 10$$

$$c^2 = a^2 + b^2 \implies b^2 = c^2 - a^2 = 100 - 36 = 64 \implies b = 8$$

$$\frac{y^2}{36} - \frac{x^2}{64} = 1 \quad \text{المعادلة:}$$

تمارين إثرائية



$$(1) \text{ معادلات الخطوط المقاربة للقطع الزائد: } \frac{x^2}{49} - \frac{y^2}{9} = 1$$

$$y = \pm \frac{3}{7}x \quad \therefore y = \pm \frac{b}{a}x$$

$$(2) a = 10 \quad b = 7$$

$$c^2 = a^2 - b^2 = 100 - 49 = 51 \implies c = \sqrt{51}$$

$$\therefore F_1(-\sqrt{51}, 0), F_2(\sqrt{51}, 0)$$

$$(3) m = 0 \implies y^2 - x = 0$$

$$y^2 = x$$

معادلة قطع مكافئ رأسه نقطة الأصل

$$4p = 1 \implies p = \frac{1}{4}$$

$$F\left(\frac{1}{4}, 0\right) \quad \text{البؤرة:}$$

$$\text{معادلة الدليل } x = -\frac{1}{4}$$

$$(4) x^2 - 5y^2 + 7 = 0 \implies x^2 - 5y^2 = -7 \implies 5y^2 - x^2 = 7$$

$$\frac{y^2}{7} - \frac{x^2}{5} = 1$$

إذاً هي معادلة قطع زائد مركزه نقطة الأصل.

$$a^2 = \frac{7}{5} \implies a = \sqrt{\frac{7}{5}}$$

$$b^2 = 7 \implies b = \sqrt{7}$$

$$\text{الرأسان: } A_1\left(0, -\sqrt{\frac{7}{5}}\right); A_2\left(0, \sqrt{\frac{7}{5}}\right)$$

$$\text{معادلتا الخطين المقاربتين: } y = \pm \frac{\sqrt{\frac{7}{5}}}{\sqrt{7}}x = \pm \frac{\sqrt{7}}{\sqrt{5} \times \sqrt{7}}x = \pm \frac{\sqrt{5}}{5}x$$

$$c^2 = a^2 + b^2 \Rightarrow c^2 = \frac{7}{5} + 7 = \frac{42}{5} \Rightarrow c = \sqrt{\frac{42}{5}}$$

$$F_1\left(0, -\sqrt{\frac{42}{5}}\right); F_2\left(0, \sqrt{\frac{42}{5}}\right)$$

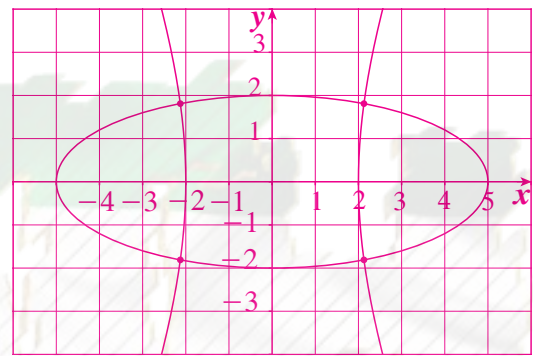
$$y = \pm \frac{a^2}{c} = \pm \frac{7}{\frac{5\sqrt{42}}{\sqrt{5}}} = \pm \frac{\sqrt{210}}{30} \quad \text{معادلنا الدليلين:}$$

$$(5) \text{ (a) } \frac{x^2}{4} - \frac{y^2}{25} = 1$$

إذا هي معادلة قطع زائد مركزه نقطة الأصل.

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

إذا هي معادلة قطع ناقص مركزه نقطة الأصل.



(b)

يبين الشكل وجود 4 نقاط تقاطع بين المنحنيين.

$$(c) \frac{x^2}{4} = 1 + \frac{y^2}{25}$$

$$x^2 = 4\left(1 + \frac{y^2}{25}\right)$$

$$\frac{x^2}{25} = 1 - \frac{y^2}{4}$$

$$x^2 = 25\left(1 - \frac{y^2}{4}\right)$$

$$\Rightarrow 4\left(1 + \frac{y^2}{25}\right) = 25\left(1 - \frac{y^2}{4}\right)$$

$$\Rightarrow 4 + \frac{4}{25}y^2 = 25 - \frac{25}{4}y^2 \Rightarrow y^2\left(\frac{4}{25} + \frac{25}{4}\right) = 25 - 4$$

$$\frac{641}{100}y^2 = 21$$

$$y^2 = \frac{2100}{641}$$

$$y = \pm 10\sqrt{\frac{21}{641}}$$

$$x^2 = \frac{2900}{641}$$

$$x = \pm 10\sqrt{\frac{29}{641}}$$

يوجد 4 نقاط تقاطع بين المنحنيين.

$$(6) e = \frac{7}{5}, \frac{7}{5} > 1$$

إذا قطع زائد.

$$\frac{7}{5} = \frac{c}{a} \Rightarrow 7a = 5c \Rightarrow a = \frac{5}{7}c$$

$$\frac{25}{7} = \frac{a^2}{c} = \frac{\frac{25}{49}c^2}{c} = \frac{25}{49}c \Rightarrow c = 7 \Rightarrow a = 5$$

$$c^2 = a^2 + b^2 \Rightarrow b^2 = c^2 - a^2 = 7^2 - 5^2 = 24$$

$$\frac{y^2}{25} - \frac{x^2}{24} = 1 \quad \text{معادلة القطع الزائد:}$$

$$(7) e = \frac{5}{7}, \frac{5}{7} < 1$$

إذا إنه قطع ناقص

$$c = 5$$

$$\frac{c}{a} = \frac{5}{7}; \frac{5}{a} = \frac{5}{7} \Rightarrow a = 7$$

$$a^2 = b^2 + c^2 \Rightarrow b^2 = a^2 - c^2 \Rightarrow b^2 = 49 - 25 \Rightarrow b^2 = 24$$

$$\frac{x^2}{49} + \frac{y^2}{24} = 1 \quad \text{المعادلة:}$$

(8) الخط المقارب $y = \frac{b}{a}x$ يمر بالنقطة $A(3, 5)$ فيكون:

$$5 = \frac{b}{a}(3) \Rightarrow \frac{b}{a} = \frac{5}{3} \Rightarrow a = \frac{3}{5}b$$

$$c^2 = a^2 + b^2 \Rightarrow 34 = \frac{9b^2}{25} + b^2 \Rightarrow 34 = \frac{34b^2}{25} \Rightarrow b^2 = 25 \Rightarrow b = 5$$

$$a = \frac{3}{5}(5) = 3$$

$$\frac{x^2}{9} - \frac{y^2}{25} = 1 \quad \text{فتكون معادلة القطع الزائد:}$$

$$(9) \frac{a}{b} = 2, c = \sqrt{5}, a = 2b$$

$$c^2 = a^2 + b^2 \Rightarrow 5 = b^2 + 4b^2 \Rightarrow 5 = 5b^2 \Rightarrow b^2 = 1 \Rightarrow b = 1$$

$$a = 2b \Rightarrow a = 2 \quad \text{ولكن:}$$

$$\frac{y^2}{4} - x^2 = 1 \quad \text{لذا معادلة القطع الزائد هي:}$$

$$(10) a^2 = 25 \Rightarrow a = 5$$

$$b^2 = 9 \Rightarrow b = 3$$

$$a^2 = b^2 + c^2 \Rightarrow c^2 = b^2 - a^2 = 25 - 9 = 16 \Rightarrow c = 4$$

$$e = \frac{c}{a} = \frac{4}{5} \quad \text{الاختلاف المركزي:}$$

$$F_1(0, -4), F_2(0, 4) \quad \text{البؤرتان:}$$

$$x = \pm \frac{a^2}{c} = \pm \frac{25}{4} \quad \text{معادلتنا الدليلين:}$$

$$(11) 8y^2 - 25x^2 = 200 \implies \frac{y^2}{25} - \frac{x^2}{8} = 1$$

$$a^2 = 25 \implies a = 5$$

$$b^2 = 8 \implies b = 2\sqrt{2}$$

$$c^2 = a^2 + b^2 = 25 + 8 = 33 \implies c = \sqrt{33}$$

$$e = \frac{c}{a} = \frac{\sqrt{33}}{5} \quad \text{الاختلاف المركزي:}$$

$$F_1(0, -\sqrt{33}); F_2(0, \sqrt{33}) \quad \text{البؤرتان:}$$

$$y = \pm \frac{a^2}{c} = \pm \frac{25}{\sqrt{33}} = \pm \frac{25\sqrt{33}}{33} \quad \text{معادلتا الدليلين:}$$

$$y = \pm \frac{a}{b}x = \pm \frac{5\sqrt{2}}{4}x \quad \text{معادلتا الخططين المقاربتين:}$$

$$(12) x^2 = -2y$$

$$4p = -2 \implies p = -\frac{1}{2}$$

$$e = 1 \quad \text{الاختلاف المركزي:}$$

$$F\left(0, -\frac{1}{2}\right) \quad \text{البؤرة:}$$

$$y = \frac{1}{2} \quad \text{معادلة الدليل:}$$

$$(13) y^2 = -x$$

$$4p = -1 \implies p = -\frac{1}{4}$$

$$e = 1 \quad \text{الاختلاف المركزي:}$$

$$F\left(-\frac{1}{4}, 0\right) \quad \text{البؤرة:}$$

$$x = \frac{1}{4} \quad \text{معادلة الدليل:}$$

$$(14) 5x^2 - 9y^2 = 45 \implies \frac{x^2}{9} - \frac{y^2}{5} = 1$$

$$a^2 = 9 \implies a = 3$$

$$b^2 = 5 \implies b = \sqrt{5}$$

$$c^2 = a^2 + b^2 = 9 + 5 = 14 \implies c = \sqrt{14}$$

$$e = \frac{c}{a} = \frac{\sqrt{14}}{3} \quad \text{الاختلاف المركزي:}$$

$$F_1(-\sqrt{14}, 0); F_2(\sqrt{14}, 0) \quad \text{البؤرتان:}$$

$$x = \pm \frac{a^2}{c} = \pm \frac{9\sqrt{14}}{14} \quad \text{معادلتا الدليلين:}$$

$$y = \pm \frac{b}{a}x = \pm \frac{\sqrt{5}}{3}x \quad \text{معادلتا الخططين المقاربتين:}$$

المجموعة A تمارين مقالية

(1) (a) فضاء العينة: $(S) = \{(H, T), (T, T), (T, H), (H, H)\}$ عدد عناصره: $n(S) = 4$

(b) $X \in \{0, 1, 2\}$

(c) $P(X = 0) = \frac{1}{4}$

$$P(X = 1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(X = 2) = \frac{1}{4}$$

(d) دالة التوزيع الاحتمالي للمتغير العشوائي X :

x	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(2) (a) $X = \{0, 1, 2, 3\}$

متغير عشوائي متقطع.

(b) $Y = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$

متغير عشوائي متقطع.

(c) $Z = \{1, 2, 3, 4\}$

متغير عشوائي متقطع.

(3) $k = 1 - (0.1 + 0.3 + 0.2 + 0.3) = 0.1$

(4) $f(2) = 1 - (0.1 + 0.4 + 0.2) = 0.3$

دالة التوزيع الاحتمالي f للمتغير العشوائي X :

x	1	2	3	4
$f(x)$	0.1	0.3	0.4	0.2

(5) (a) عدد عناصر فضاء العينة: $n(S) = {}_{10}C_5 = 252$

(b) $X \in \{0, 1, 2, 3, 4\}$

(c) $P(X = 0) = \frac{{}_6C_5 \times {}_4C_0}{252} = \frac{1}{42}$

$$P(X = 1) = \frac{{}_6C_4 \times {}_4C_1}{252} = \frac{5}{21}$$

$$P(X = 2) = \frac{{}_6C_3 \times {}_4C_2}{252} = \frac{10}{21}$$

$$P(X = 3) = \frac{{}_6C_2 \times {}_4C_3}{252} = \frac{5}{21}$$

$$P(X = 4) = \frac{{}_6C_1 \times {}_4C_4}{252} = \frac{1}{42}$$

(d) دالة التوزيع الاحتمالي f للمتغير العشوائي X :

x	0	1	2	3	4
$f(x)$	$\frac{1}{42}$	$\frac{5}{21}$	$\frac{10}{21}$	$\frac{5}{21}$	$\frac{1}{42}$

(6) $\mu = 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.1 = 1.4$

إذاً، التوقع: $(\mu) = 1.4$

(7) (a) $\mu = 7 \times \frac{1}{8} + 8 \times \frac{3}{8} + 9 \times \frac{3}{8} + 10 \times \frac{1}{8} = \frac{17}{2}$

إذاً، التوقع: $(\mu) = \frac{17}{2}$

(b) $\sigma^2 = 49 \times \frac{1}{8} + 64 \times \frac{3}{8} + 81 \times \frac{3}{8} + 100 \times \frac{1}{8} - \left(\frac{17}{2}\right)^2 = 0.75$

إذاً، التباين: $(\sigma^2) = 0.75$

(c) $\sigma = \sqrt{0.75} = 0.866$

إذاً، الانحراف المعياري: $(\sigma) = 0.866$

(8) $F(0) = P(X \leq 0) = 0.2$

$$F(1) = P(X \leq 1) = P(X < 1) + P(X = 1) = 0.2 + 0.15 = 0.35$$

$$F(2) = P(X \leq 2) = P(X < 2) + P(X = 2) = 0.2 + 0.15 + 0.1 = 0.45$$

$$F(3) = P(X \leq 3) = P(X < 3) + P(X = 3) = 0.2 + 0.15 + 0.1 + 0.25 = 0.7$$

$$F(3.5) = P(X \leq 3.5) = P(X < 3) + P(X = 3) = 0.2 + 0.15 + 0.1 + 0.25 = 0.7$$

$$F(4) = P(X \leq 4) = P(X < 4) + P(X = 4) = 0.2 + 0.15 + 0.1 + 0.25 + 0.3 = 1$$

$$F(5) = P(X \leq 5) = P(X < 5) + P(X = 5) = 1$$

(9) (a) $P(-1 < X < 5) = F(5) - F(-1) = 0.7 - 0.1 = 0.6$

(b) $P(3 \leq X < 7) = F(7) - F(3) = 1 - 0.45 = 0.55$

(c) $P(X > 3) = 1 - P(X \leq 3) = 1 - F(3) = 1 - 0.45 = 0.55$

(10) (a) $P(X = 0) = {}_8C_0 \times 0.3^0 \times (1 - 0.3)^8 = 0.0576$

(b) $P(2 < X \leq 5) = P(X = 3) + P(X = 4) + P(X = 5)$

$$= {}_8C_3 \times 0.3^3 \times 0.7^5 + {}_8C_4 \times 0.3^4 \times 0.7^4 + {}_8C_5 \times 0.3^5 \times 0.7^3 = 0.437$$

(11) (a) $P(X = 0) = {}_{10}C_0 \times 0.5^0 \times 0.5^{10} = 9.766 \cdot 10^{-4}$

(b) $P(2 < X \leq 4) = P(X = 3) + P(X = 4)$
 $= {}_{10}C_3 \times 0.5^3 \times 0.5^7 + {}_{10}C_4 \times 0.5^4 \times 0.5^6 = 0.322$

(12) $n = 100$, $p = 0.03$

$\mu = n p = 100 \times 0.03 = 3$

إذاً، التوقع: $(\mu) = 3$

$\sigma^2 = n p(1 - p) = 100 \times 0.03 \times 0.97 = 2.91$

إذاً، التباين: $(\sigma^2) = 2.91$

$\sigma = \sqrt{2.91} = 1.7059$

إذاً، الانحراف المعياري: $(\sigma) = 1.7059$

(13) $n = 12$, $p = 0.5$

$\mu = n p = 12 \times 0.5 = 6$

إذاً، التوقع: $(\mu) = 6$

$\sigma^2 = n p(1 - p) = 12 \times 0.5 \times 0.5 = 3$

إذاً، التباين: $(\sigma^2) = 3$

$\sigma = \sqrt{3} = 1.732$

إذاً، الانحراف المعياري: $(\sigma) = 1.732$

المجموعة B تمارين موضوعية

- | | | | | | |
|----------|----------|----------|----------|----------|----------|
| (1) (b) | (2) (b) | (3) (a) | (4) (b) | (5) (b) | (6) (a) |
| (7) (b) | (8) (b) | (9) (b) | (10) (c) | (11) (b) | (12) (a) |
| (13) (d) | (14) (d) | (15) (a) | (16) (b) | (17) (c) | (18) (c) |
| (19) (b) | (20) (c) | (21) (b) | | | |

المجموعة A تمارين مقالية

(1) (a) $P(0 \leq X \leq 5) = 5 \times \frac{1}{5} = 1$

(b) $P(X = 3) = 0$

(c) $P(X \leq 2) = 2 \times \frac{1}{5} = \frac{2}{5}$

(d) $P(X > 2) = 3 \times \frac{1}{5} = \frac{3}{5}$

(2) (a) $P(2 \leq X \leq 4) = 2 \times \frac{1}{2} = 1$

(b) $P(X \geq 2.5) = (4 - 2.5) \times \frac{1}{2} = \frac{3}{4}$

(3) (a) $x = 3 \quad \therefore y = \frac{6}{9} = \frac{2}{3}$

$P(0 \leq X \leq 3) = \frac{1}{2} \times 3 \times \frac{2}{3} = 1$

(b) $x = 1 \quad \therefore y = \frac{2}{9}$

$P(X < 1) = \frac{1}{2} \times 1 \times \frac{2}{9} = \frac{1}{9}$

(c) $P(X \geq 1) = 1 - P(X < 1) = 1 - \frac{1}{9} = \frac{8}{9}$

(4) (a) المساحة تحت المنحنى (وهو منطقة مستطيلة)

$\frac{1}{6} \times (5 - (-1)) = 6 \times \frac{1}{6} = 1$

∴ الدالة هي كثافة احتمال.

(b) لإثبات أن الدالة f تتبع التوزيع الاحتمالي المنتظم يجب أن تكون الدالة f على الصورة:

$$f(x) = \begin{cases} \frac{1}{b-a} & : a \leq x \leq b \\ 0 & : \text{في ما عدا ذلك} \end{cases}$$

$$a = -1, b = 5$$

$$f(x) = \begin{cases} \frac{1}{5 - (-1)} = \frac{1}{6} & : -1 \leq x \leq 5 \\ 0 & : \text{في ما عدا ذلك} \end{cases}$$

إذاً f هي دالة توزيع احتمالي منتظم.

(c) $P(0 < X \leq 3) = 3 \times \frac{1}{6} = \frac{1}{2}$

(d) $\mu = \frac{a+b}{2} = \frac{5-1}{2} = 2$

إذاً، التوقع: $(\mu) = 2$

$$\sigma^2 = \frac{(b-a)^2}{12} = \frac{(5-(-1))^2}{12} = \frac{36}{12} = 3$$

إذاً، التباين $(\sigma^2) = 3$

(5) (a) لإثبات أن الدالة f تتبع التوزيع الاحتمالي المنتظم يجب أن تكون الدالة f على الصورة:

$$f(x) = \begin{cases} \frac{1}{b-a} & : a \leq x \leq b \\ 0 & : \text{في ما عدا ذلك} \end{cases}$$

$$a = 0, b = 7$$

$$f(x) = \begin{cases} \frac{1}{7-0} = \frac{1}{7} & : 0 \leq x \leq 7 \\ 0 & : \text{في ما عدا ذلك} \end{cases}$$

إذاً f هي دالة توزيع احتمالي منتظم.

$$(b) P\left(0 \leq X \leq \frac{7}{8}\right) = \frac{7}{8} \times \frac{1}{7} = \frac{1}{8}$$

$$(c) \mu = \frac{a+b}{2} = \frac{0+7}{2} = \frac{7}{2}$$

إذاً، التوقع: $(\mu) = \frac{7}{2}$

$$\sigma^2 = \frac{(b-a)^2}{12} = \frac{(7-0)^2}{12} = \frac{49}{12}$$

إذاً، التباين: $(\sigma^2) = \frac{49}{12}$

$$(6) (a) P(z \leq 2.16) = 0.98461$$

$$(b) P(z \geq 2.51) = 1 - P(z < 2.51) = 1 - 0.99396 = 0.00604$$

$$(c) P(1.5 \leq z \leq 2.4) = P(z \leq 2.4) - P(z \leq 1.5) = 0.99180 - 0.93319 = 0.05861$$

$$(7) (a) P(z \leq -0.64) = 0.26109$$

$$(b) P(-1.7 \leq z \leq 2.85) = P(z \leq 2.85) - P(z \leq -1.7)$$

$$= 0.99781 - 0.04457 = 0.95324$$

$$(c) P(-1.23 \leq z \leq 0.68) = P(z \leq 0.68) - P(z \leq -1.23)$$

$$= 0.75175 - 0.10935 = 0.6424$$

المجموعة B تمارين موضوعية

- | | | | | | |
|----------|----------|----------|----------|----------|----------|
| (1) (a) | (2) (b) | (3) (b) | (4) (b) | (5) (a) | (6) (a) |
| (7) (a) | (8) (b) | (9) (a) | (10) (b) | (11) (d) | (12) (b) |
| (13) (a) | (14) (d) | (15) (c) | (16) (d) | (17) (c) | |

اختبار الوحدة الثامنة

(1) $f(5) = 1 - (0.3 + 0.2 + 0.1) = 0.4$

دالة التوزيع الاحتمالي f للمتغير العشوائي X :

x	2	3	4	5
$f(x)$	0.3	0.2	0.1	0.4

(2) (a) $n(S) = {}_8C_4 = 70$

(b) $X \in \{0, 1, 2, 3\}$

(c) $P(X = 0) = \frac{{}_5C_4}{{}_7C_4} = \frac{1}{14}$

$P(X = 1) = \frac{{}_5C_3 \times {}_3C_1}{{}_7C_4} = \frac{3}{7}$

$P(X = 2) = \frac{{}_5C_2 \times {}_3C_2}{{}_7C_4} = \frac{3}{7}$

$P(X = 3) = \frac{{}_5C_1 \times {}_3C_3}{{}_7C_4} = \frac{1}{14}$

(d) دالة التوزيع الاحتمالي f للمتغير العشوائي X :

x	0	1	2	3
$f(x)$	$\frac{1}{14}$	$\frac{3}{7}$	$\frac{3}{7}$	$\frac{1}{14}$

(3) (a) $\mu = 3 \times \frac{2}{11} + 4 \times \frac{5}{11} + 5 \times \frac{3}{11} + 6 \times \frac{1}{11} = \frac{47}{11}$

إذاً، التوقع: $(\mu) = \frac{47}{11}$

(b) $\sigma^2 = 9 \times \frac{2}{11} + 16 \times \frac{5}{11} + 25 \times \frac{3}{11} + 36 \times \frac{1}{11} - \left(\frac{47}{11}\right)^2 = \frac{90}{121}$

إذاً، التباين: $(\sigma^2) = \frac{90}{121}$

(c) $\sigma = \sqrt{\frac{90}{121}} = \frac{3}{11}\sqrt{10}$

إذاً، الانحراف المعياري: $(\sigma) = \frac{3}{11}\sqrt{10}$

(4) $F(1) = p(X \leq 1) = 0$

$F(2) = p(X \leq 2) = p(X < 2) + p(X = 2) = 0.14$

$F(3) = p(X \leq 3) = p(X = 3) + p(X < 3) = p(X = 3) + p(X = 2) = 0.3$

$F(3.5) = p(X \leq 3.5) = p(X = 3) + p(X < 3) = p(X = 3) + p(X = 2) = 0.3$

$F(4) = p(X \leq 4) = p(X = 4) + p(X < 4) = p(X = 4) + p(X = 3) + p(X = 2) = 0.65$

$F(5) = p(X \leq 5) = p(X = 5) + p(X < 5) = p(X = 5) + p(X = 4) + p(X = 3) + p(X = 2) = 0.8$

$F(6) = p(X \leq 6) = p(X = 6) + p(X < 6) = p(X = 6) + p(X = 5) + p(X = 4) + p(X = 3) + p(X = 2) = 1$

$F(7) = p(X \leq 7) = p(X = 7) + p(X < 7) = p(X = 6) + p(X = 5) + p(X = 4) + p(X = 3) + p(X = 2) = 1$

(5) $n = 1250$, $p = 0.04$

(a) $\mu = np = 1250 \times 0.04 = 50$

إذاً، التوقع: $(\mu) = 50$

(b) $\sigma^2 = np(1-p) = 1250 \times 0.04 \times 0.96 = 48$

إذاً، التباين: $(\sigma^2) = 48$

(c) $\sigma = \sqrt{48} = 4\sqrt{3}$

إذاً، الانحراف المعياري: $(\sigma) = 4\sqrt{3}$

(6) (a) $P(0 \leq X \leq 3) = 3 \times \frac{1}{5} = \frac{3}{5}$

(b) $P(-2 \leq X \leq 0) = 2 \times \frac{1}{5} = \frac{2}{5}$

(c) $P(X = 2) = 0$

(d) $P(-1 \leq X \leq 2) = (2 - (-1)) \times \frac{1}{5} = \frac{3}{5}$

(7) (a) $x = \frac{1}{3} \therefore y = \frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$

$$P\left(0 \leq X \leq \frac{1}{3}\right) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{2} = \frac{1}{4}$$

(b) $P\left(X \geq \frac{1}{3}\right) = 1 - P\left(X < \frac{1}{3}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

(8) (a) المساحة تحت منحنى الدالة f هي: $(5 - (-3)) \times \frac{1}{8} = 8 \times \frac{1}{8} = 1$
∴ الدالة f هي دالة كثافة احتمال.

(b) $P(-1 \leq x \leq 3) = (3 - (-1)) \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

(c) $\mu = \frac{a+b}{2} = \frac{-3+5}{2} = 1$

إذاً، التوقع: $(\mu) = 1$

$$\sigma^2 = \frac{(b-a)^2}{12} = \frac{(5 - (-3))^2}{12} = \frac{64}{12} = \frac{16}{3}$$

إذاً، التباين: $(\sigma^2) = \frac{16}{3}$

(9) (a) $P(z \leq 2.24) = 0.98745$

(b) $P(z \geq 1.52) = 1 - P(z < 1.52) = 1 - 0.93574 = 0.06426$

(c) $P(1.4 \leq z \leq 2.6) = P(x \leq 2.6) - P(x \leq 1.4) = 0.99534 - 0.91924 = 0.0761$

(10) (a) $x_1 = 30 \therefore z_1 = \frac{x_1 - \mu}{\sigma} = \frac{30 - 40}{8} = -\frac{5}{4} = -1.25$

$$x_2 = 65 \therefore z_2 = \frac{x_2 - \mu}{\sigma} = \frac{65 - 40}{8} = \frac{25}{8} = 3.125$$

$$\begin{aligned} P(30 < X < 65) &= P(-0.125 < z < 3.125) = P(z < 3.125) - P(z < -1.25) \\ &= \frac{0.99910 + 0.99913}{2} - 0.10565 = 0.893465 \end{aligned}$$

$$(b) X = 45 \quad \therefore z = \frac{X - \mu}{\sigma} = \frac{45 - 40}{8} = \frac{5}{8} = 0.625$$

$$P(X \geq 45) = 1 - P(X < 45) = 1 - P(z < 0.625) = 1 - \frac{0.73237 + 0.73565}{2} \\ = 1 - 0.73401 = 0.26599$$

$$(11) K = 1 - (0.16 + 0.24 + 0.15 + 0.2) = 0.25$$

$$(12) (a) P(z \leq 1.45) = 0.92647$$

$$(b) P(z > 0.27) = 1 - P(z \leq 0.27) = 1 - 0.60642 = 0.39358$$

$$(c) P(-1.32 \leq z \leq 1.75) = P(z \leq 1.75) - P(z \leq -1.32) = 0.95994 - 0.09342 = 0.86652$$

$$(d) P(-2.87 \leq z \leq -1.42) = P(x \leq -1.42) - P(x \leq -2.87) = 0.07780 - 0.00205 = 0.07575$$

تمارين إثرائية

$$(1) \sigma^2 = 25 \quad \therefore \sigma = 5$$

$$(a) x = 55 \quad \therefore z = \frac{x - \mu}{\sigma} = \frac{55 - 55}{5} = 0$$

$$P(X > 55) = 1 - P(X \leq 55) = 1 - P(z \leq 0) = 1 - 0.5 = 0.5$$

$$(b) x = 50 \quad \therefore z = \frac{x - \mu}{\sigma} = \frac{50 - 55}{5} = -\frac{5}{5} = -1$$

$$P(X < 50) = P(z < -1) = 0.15866$$

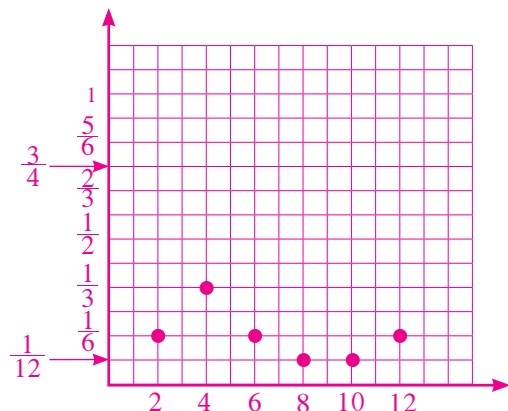
$$(c) x_1 = 30 \quad \therefore z_1 = \frac{x_1 - \mu}{\sigma} = \frac{30 - 55}{5} = -5$$

$$x_2 = 40 \quad \therefore z_2 = \frac{x_2 - \mu}{\sigma} = \frac{40 - 55}{5} = -3$$

$$P(30 < X < 40) = P(-5 < z < -3) = P(z < -3) - P(z < -5) \\ = 0.00135 - 0 = 0.00135$$

$$(2) (a) K = 1 - \left(\frac{1}{6} + \frac{1}{3} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6} \right) = \frac{1}{6}$$

(b)



$$(c) F(2) = P(X \leq 2) = P(X = 2) = \frac{1}{6}$$

$$F(4) = P(X \leq 4) = P(X = 2) + P(X = 4) = \frac{1}{2}$$

$$F(6) = P(X \leq 6) = P(X = 2) + P(X = 4) + P(X = 6) = \frac{2}{3}$$

$$F(8) = P(X \leq 8) = P(X = 2) + P(X = 4) + P(X = 6) + P(X = 8) = \frac{3}{4}$$

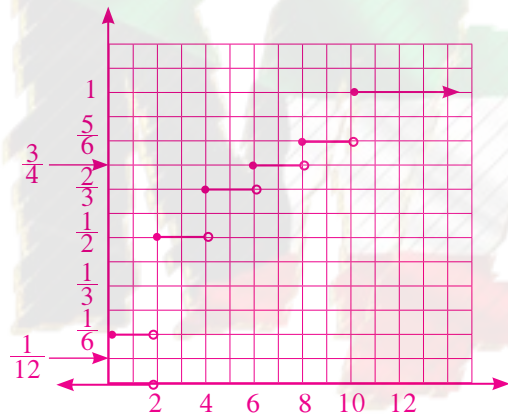
$$F(10) = P(X \leq 10) = P(X = 2) + P(X = 4) + P(X = 6) + P(X = 8) + P(X = 10) = \frac{5}{6}$$

$$F(12) = P(X \leq 12) = P(X = 2) + P(X = 4) + P(X = 6) + P(X = 8) + P(X = 10) + P(X = 12) = 1$$

جدول التوزيع التراكمي F للمتغير العشوائي المتقطع X :

x	2	4	6	8	10	12
$F(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	1

(d)



$$(3) \mu = 14 \quad \sigma = \sqrt{1} = 1$$

$$(a) x = 15 \therefore z = \frac{x - \mu}{\sigma} = 15 - 14 = 1$$

$$P(X > 15) = P(z > 1) = 1 - P(z \leq 1) = 1 - 0.84134 = 0.15866$$

$$(b) x = 11 \therefore z = \frac{x - \mu}{\sigma} = 11 - 14 = -3$$

$$P(X < 11) = P(z < -3) = 0.00135$$

$$(c) x_1 = 13 \therefore z_1 = \frac{x_1 - \mu}{\sigma} = 13 - 14 = -1$$

$$x_2 = 15 \therefore z_2 = \frac{x_2 - \mu}{\sigma} = 15 - 14 = 1$$

$$P(13 < X < 15) = P(-1 < z < 1) = P(z < 1) - P(z < -1) \\ = 0.84134 - 0.15866 = 0.68268$$

$$(4) (a) P\left(\frac{1}{2} \leq X \leq \frac{3}{2}\right) = P\left(X \leq \frac{3}{2}\right) - P\left(X \leq \frac{1}{2}\right) = f\left(\frac{3}{2}\right) - f\left(\frac{1}{2}\right)$$

$$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right) = 3$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) = 1$$

$$P\left(\frac{1}{2} \leq X \leq \frac{3}{2}\right) = \frac{1}{2} \times \frac{3}{2} \times 3 - \frac{1}{2} \times \frac{1}{2} \times 1 = 2$$

$$(b) P\left(X \geq \frac{1}{2}\right) = 1 - P\left(X < \frac{1}{2}\right) = 1 - \left(\frac{1}{2} \times \frac{1}{2} \times 1\right) = \frac{3}{4}$$

$$(5) n = 7, p = \frac{1}{2}$$

$$(a) P(X = 5) = {}_7C_5 \times 0.5^5 \times 0.5^2 = 0.164$$

$$(b) P(X > 0) = 1 - P(X \leq 0) = 1 - P(X = 0) = 1 - {}_7C_0 \times 0.5^0 \times 0.5^7 = 0.992$$

$$(c) P(X = 0) + P(X = 1) = 7.8125 \cdot 10^{-3} + {}_7C_1 \times 0.5^1 \times 0.5^6 = 0.0625$$

$$(6) (a) P(z \leq 2.65) = 0.99598$$

$$(b) P(-2.85 \leq z \leq -1.96) = P(z \leq -1.96) - P(z \leq -2.85) = 0.025 - 0.00219 = 0.02281$$

$$(c) P(z \geq 1.56) = 1 - P(z < 1.56) = 1 - 0.94062 = 0.05938$$

$$(7) (a) \mu = 1 \times \frac{1}{6} + 2 \times \frac{1}{4} + 3 \times \frac{1}{3} + 4 \times \frac{1}{12} + 5 \times \frac{1}{6} = \frac{17}{6}$$

إذا، التوقع: $(\mu) = \frac{17}{6}$

$$(b) \sigma^2 = 1 \times \frac{1}{6} + 4 \times \frac{1}{4} + 9 \times \frac{1}{3} + 16 \times \frac{1}{12} + 25 \times \frac{1}{6} - \left(\frac{17}{6}\right)^2 = \frac{59}{36}$$

إذا، التباين: $(\sigma^2) = \frac{59}{36}$

$$(c) \sigma = \sqrt{\frac{59}{36}} = \frac{\sqrt{59}}{6}$$

إذا، الانحراف المعياري: $(\sigma) = \frac{\sqrt{59}}{6}$

$$(8) F(2) = P(X \leq 2) = 0$$

$$F(3) = P(X \leq 3) = P(X < 3) + P(X = 3) = P(X = 3) = 0.17$$

$$F(4) = P(X \leq 4) = P(X < 4) + P(X = 4) = P(X = 3) + P(X = 4) = 0.41$$

$$F(4.5) = P(X \leq 4.5) = P(X < 4) + P(X = 4) = P(X = 3) + P(X = 4) = 0.41$$

$$F(5) = P(x \leq 5) = P(X < 5) + P(X = 5) = P(X = 3) + P(X = 4) + P(X = 5) = 0.64$$

$$F(6) = P(X \leq 6) = P(X < 6) + P(X = 6) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) = 1$$

$$F(6.5) = P(X \leq 6.5) = P(X < 6) + P(X = 6) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) = 1$$

