تمرن (5-5) التكامل بالتجزيء

Integration by Parts

المجموعة A تمارين مقالية

في التمارين (1-14)، أوجد التكامل.

(1)

$$\int x\cos(3x)\,dx$$

$u = x,\qquad dv = \cos(3x)\,dx$

$du = dx,\qquad v = \frac{1}{3}\sin(3x)$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\cos(3x)\,dx = \frac{1}{3}x\sin(3x) - \int \frac{1}{3}\sin(3x)\,dx$$

$$= \frac{1}{3}x\sin(3x) + \frac{1}{9}\cos(3x) + C$$

(2)

$$\int x\sin(5x)\,dx$$

$u = x,\qquad dv = \sin(5x)\,dx$

$du = dx,\qquad v = -\frac{1}{5}\cos(5x)$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\sin(5x)\,dx = -\frac{1}{5}x\cos(5x) + \int \frac{1}{5}\cos(5x)\,dx$$

$$= -\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C$$

(3)

$$\int xe^{x-3}\,dx$$

$u = x,\qquad dv = e^{x-3}dx$

$du = dx,\qquad v = e^{x-3}$

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^{x-3}\,dx = xe^{x-3} - \int e^{x-3}\,dx$$

$$= xe^{x-3} - e^{x-3} + C = (x-1)e^{x-3} + C$$

(4)

$$\int (x-5)e^{x-5}\,dx$$

$u = x-5,\qquad dv = e^{x-5}dx$

$du = dx,\qquad v = e^{x-5}$

$$\int u\,dv = uv - \int v\,du$$

$$\int (x-5)e^{x-5}\,dx = (x-5)e^{x-5} - \int e^{x-5}\,dx$$

$$= (x-5)e^{x-5} - e^{x-5} + C = (x-6)e^{x-5} + C$$

(5)

$$\int \ln \sqrt[4]{x}\, dx$$

$$u = \ln \sqrt[4]{x} = \frac{1}{4}\ln x\,, \qquad dv = dx$$

$$du = \frac{1}{4x}dx, \qquad v = x$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \ln \sqrt[4]{x}\, dx = x \ln \sqrt[4]{x} - \int \frac{1}{4}dx = x \ln \sqrt[4]{x} - \frac{1}{4}x + C$$

(6)

$$\int \ln(2x-1)\, dx$$

$$u = \ln(2x-1), \qquad dv = dx$$

$$du = \frac{2}{2x-1}dx, \qquad v = x$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \ln(2x-1)\, dx = x\ln(2x-1) - \int \frac{2x}{2x-1}dx$$

$$= x\ln(2x-1) - \int \frac{2x-1+1}{2x-1}dx$$

$$= x\ln(2x-1) - \int \left(1 + \frac{1}{2x-1}\right)dx$$

$$= x\ln(2x-1) - x - \frac{1}{2}\ln|2x-1| + C$$

$$= x\ln(2x-1) - x - \frac{1}{2}\ln(2x-1) + C$$

(7)

$$\int (2x+1)\ln(x+1)\,dx$$

$u = \ln(x+1),$ $\quad\quad$ $dv = (2x+1)dx$

$du = \frac{1}{x+1}dx,$ $\quad\quad$ $v = x^2 + x$

$$\int u\,dv = uv - \int v\,du$$

$$\int (2x+1)\ln(x+1)\,dx = (x^2+x)\ln(x+1) - \int \frac{x^2+x}{x+1}dx$$

$$= (x^2+x)\ln(x+1) - \int \frac{x(x+1)}{x+1}dx$$

$$= (x^2+x)\ln(x+1) - \int x\,dx$$

$$= (x^2+x)\ln(x+1) - \frac{x^2}{2} + C$$

(8)

$$\int \frac{\ln x}{x^2}dx$$

$u = \ln x\,,$ $\quad\quad$ $dv = \frac{1}{x^2}dx = x^{-2}dx$

$du = \frac{1}{x}dx,$ $\quad\quad$ $v = -\frac{1}{x}$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}\ln x + \int x^{-2}\,dx = -\frac{1}{x}\ln x - \frac{1}{x} + C$$

(9)

$$\int \frac{\ln x}{\sqrt[3]{x}}\,dx$$

$u = \ln x\,,$ $\qquad\qquad dv = \frac{1}{\sqrt[3]{x}}\,dx = x^{-\frac{1}{3}}dx$

$du = \frac{1}{x}\,dx,$ $\qquad\qquad v = \frac{3}{2}x^{\frac{2}{3}}$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{\ln x}{\sqrt[3]{x}}\,dx = \frac{3}{2}x^{\frac{2}{3}}\ln x - \int \frac{3}{2}x^{-\frac{1}{3}}dx = \frac{3}{2}x^{\frac{2}{3}}\ln x - \frac{9}{4}x^{\frac{2}{3}} + C$$

(10)

$$\int x^2 \ln x^2\ \,dx$$

$u = \ln x^2 = 2\ln x\,,$ $\qquad\qquad dv = x^2 dx$

$du = \frac{2}{x}\,dx,$ $\qquad\qquad v = \frac{x^3}{3}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \ln x^2\,dx = \frac{x^3}{3}\ln x^2 - \int \frac{2}{3}x^2 dx = \frac{x^3}{3}\ln x^2 - \frac{2}{9}x^3 + C$$

(11)

$$\int (x^2 - 2x)\cos x\, dx$$

$u = x^2 - 2x,\qquad dv = \cos x\, dx$

$du = (2x - 2)dx,\qquad v = \sin x$

$$\int u\, dv = uv - \int v\, du$$

$$\int (x^2 - 2x)\cos x\, dx = (x^2 - 2x)\sin x - \int (2x - 2)\sin x\, dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int (2x - 2)\sin x\, dx$$

$u = 2x - 2,\qquad dv = \sin x\, dx$

$du = 2dx,\qquad v = -\cos x$

$$\int (2x - 2)\sin x\, dx = -(2x - 2)\cos x + \int 2\cos x\, dx$$

$$= -(2x - 2)\cos x + 2\sin x + C \qquad (2)$$

من (2) , (1) نحصل على:

$$\int (x^2 - 2x)\cos x\, dx = (x^2 - 2x)\sin x + (2x - 2)\cos x - 2\sin x + C$$

$$= (x^2 - 2x - 2)\sin x + (2x - 2)\cos x + C$$

(12)

$$\int (x^2 + 3x)\sin x\, dx$$

$u = x^2 + 3x,\qquad dv = \sin x\, dx$

$du = (2x+3)dx, \qquad v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$\int (x^2+3x)\sin x\,dx = -(x^2+3x)\cos x + \int (2x+3)\cos x\,dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int (2x+3)\cos x\,dx$$

$u = 2x+3, \qquad dv = \cos x\,dx$

$du = 2dx, \qquad v = \sin x$

$$\int (2x+3)\cos x\,dx = (2x+3)\sin x - \int 2\sin x\,dx$$

$$= (2x+3)\sin x + 2\cos x + C \qquad (2)$$

من (2) , (1) نحصل على:

$$\int (x^2+3x)\sin x\,dx = -(x^2+3x)\cos x + (2x+3)\sin x + 2\cos x + C$$

$$= (-x^2-3x+2)\cos x + (2x+3)\sin x + C$$

(13)

$$\int x^2 e^{x+1}\,dx$$

$u = x^2, \qquad dv = e^{x+1}dx$

$du = 2xdx, \qquad v = e^{x+1}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 e^{x+1}\,dx = x^2 e^{x+1} - \int 2xe^{x+1}\,dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int 2xe^{x+1}dx$$

$u = 2x,\qquad dv = e^{x+1}dx$

$du = 2dx,\qquad v = e^{x+1}$

$$\int 2xe^{x+1}dx = 2xe^{x+1} - \int 2e^{x+1}dx = 2xe^{x+1} - 2e^{x+1} + C \qquad (2)$$

من (2) , (1) نحصل على:

$$\int x^2e^{x+1}\,dx = x^2e^{x+1} - 2xe^{x+1} + 2e^{x+1} + C$$

$$= (x^2 - 2x + 2)e^{x+1} + C$$

(14)

$$\int x^2e^{2x-3}\,dx$$

$u = x^2,\qquad dv = e^{2x-3}dx$

$du = 2xdx,\qquad v = \frac{1}{2}e^{2x-3}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2e^{2x-3}\,dx = \frac{1}{2}x^2e^{2x-3} - \int xe^{2x-3}\,dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int xe^{2x-3}dx$$

$u = x,\qquad dv = e^{2x-3}dx$

$du = dx,\qquad v = \frac{1}{2}e^{2x-3}$

$$\int xe^{2x-3}dx = \frac{1}{2}xe^{2x-3} - \int \frac{1}{2}e^{2x-3}dx = \frac{1}{2}xe^{2x-3} - \frac{1}{4}e^{2x-3} + C \qquad (2)$$

من (2) , (1) نحصل على:

$$\int x^2 e^{2x-3}\,dx = \frac{1}{2}x^2e^{2x-3} - \frac{1}{2}xe^{2x-3} + \frac{1}{4}e^{2x-3} + C$$

$$= \left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}\right)e^{2x-3} + C$$

(15)

$$\int (\ln x)^2\,dx$$

$$u = (\ln x)^2, \qquad dv = dx$$

$$du = 2(\ln x)\frac{dx}{x}, \qquad v = x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int (\ln x)^2\,dx = x\,(\ln x)^2 - \int 2\ln x\,dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int 2\ln x\,dx$$

$$u = 2\ln x, \qquad dv = dx$$

$$du = \frac{2}{x}dx, \qquad v = x$$

$$\int 2\ln x\,dx = 2x\ln x - \int 2dx = 2x\ln x - 2x + C \qquad (2)$$

من (2) , (1) نحصل على:

$$\int (\ln x)^2\,dx = x(\ln x)^2 - 2x\ln x + 2x + C$$

(16)

$$\int e^{2x}\sin x\,dx$$

$u = e^{2x},\qquad dv = \sin x\,dx$

$du = 2e^{2x}dx,\qquad v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2\int e^{2x}\cos x\;dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int e^{2x}\cos x\;dx$$

$u = e^{2x},\qquad dv = \cos x\,dx$

$du = 2e^{2x}dx,\qquad v = \sin x$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\int e^{2x}\sin x\;dx \qquad (2)$$

نعوض (2) في (1) :

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2\left(e^{2x}\sin x - 2\int e^{2x}\sin x\;dx\right)$$

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\;dx$$

$$5\int e^{2x}\sin x\;dx = -e^{2x}\cos x + 2e^{2x}\sin x$$

$$\therefore \int e^{2x}\sin x\;dx = \frac{1}{5}(-e^{2x}\cos x + 2e^{2x}\sin x)$$

(17)

$$\int \sin(\ln x)\, dx$$

$u = \sin(\ln x)\,,\qquad\qquad dv = dx$

$du = \dfrac{\cos(\ln x)}{x}\, dx,\qquad\qquad v = x$

$$\int u\, dv = uv - \int v\, du$$

$$\int \sin(\ln x)\, dx = x\sin(\ln x) - \int \cos(\ln x)\, dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int \cos(\ln x)\, dx$$

$u = \cos(\ln x)\,,\qquad\qquad dv = dx$

$du = \dfrac{-\sin(\ln x)}{x}\, dx,\qquad\qquad v = x$

$$\int \cos(\ln x)\, dx = x\cos(\ln x) + \int \sin(\ln x)\, dx \qquad (2)$$

نعوض (2) في (1) :

$$\int \sin(\ln x)\, dx = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\, dx$$

$$2\int \sin(\ln x)\, dx = x\sin(\ln x) - x\cos(\ln x) + C$$

$$\therefore \int \sin(\ln x)\, dx = \frac{1}{2} x(\sin(\ln x) - \cos(\ln x)) + C$$

المجموعة B تمارين موضوعية

1. a
2. a
3. a
4. b
5. a
6. b
7. b
8. d
9. b
10. b
11. c